

Transactions

of the

A.S.M.E.

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Turbine Discharge Metering at the Safe Harbor Hydroelectric Development

By J. M. MOUSSON,¹ BALTIMORE, MD.

This paper discusses the suitability, calibration, and reliability of certain piezometer systems installed in low-head units of high capacity. An account is also given of a research to determine and develop a suitable type of flowmeter to be operated by the differential pressure from these piezometer systems for continuous integration, indication, and graphic recording of unit and plant discharges. The type of equipment installed is presented in detail, as well as its adaptation as an automatic guide to operation, resulting in appreciable benefits through higher operating efficiencies.

INTRODUCTION

ALTHOUGH continuous automatic accounting of unit discharge is not new, several recent improvements and developments have entirely changed the aspect of desirability for apparatus of this kind, as many of the shortcomings of earlier installations, limiting their usefulness, have been successfully overcome.

While, in some plants, automatic water accounting has been carried out for years, the necessary equipment has often been regarded as a luxury, particularly, as its sole purpose was usually confined to collecting runoff data at the project site to augment records of existing gaging stations or, perhaps, replace those of stations rendered inoperative in a project area due to construction of a particular plant. Since the accuracy of river gaging is essentially not very high, and decidedly lower than that required for turbine-discharge measurements for acceptance tests, it has been standard practice to keep unit- and powerhouse-draft records by means of computations based on power output. At the same time, however, it has been generally recognized that the installation of input-measuring apparatus would be highly desirable, if and when unit-discharge and station-totalizing equipment of sufficient accuracy and within economical reach were available to serve as a yardstick for plant operation, both as to proper and efficient loading of the units and to detect troubles affecting their efficiencies.

To illustrate the difficulty of the solution to this problem, it may be mentioned that, while equipment of this kind was contemplated at Safe Harbor at the outset of construction in 1930, as at that time already certain provisions had to be incorporated in the substructure, on the generator-room floor, in the conduit system, and in the control room, nevertheless, the various investigations and development work required a substantial amount of time and it was not until late in 1938 that suitable equipment was finally installed.

1—INVESTIGATIONS PURSUED

The various investigations carried out dealt not only with the exploration of the principle to be employed, but also with the

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

possible consistency, sensitivity, and suitability of various apparatus. In the first place, it had to be established that the index method, based on differential piezometer deflection, is of sufficient accuracy as a basis of continuous water measurements.

PIEZOMETER INSTALLATION

In each of the substructures of the six main units comprising the initial development, there were installed three piezometers of the Winter-Kennedy type² in the turbine scroll and two piezometers of the Peck type³ on one of the stay vanes of the speed ring, Fig. 1. While one of the Winter-Kennedy taps was placed in the high-pressure low-velocity region, the two other taps were located radially opposite thereto at the speed ring in the low-pressure high-velocity region, one just above the speed ring and the other tapped in the crown of the speed ring.

The Peck piezometer locations are shown in Fig. 2, the impact tap in the nose and the low-pressure tap in the flank of the stay vane. In the first four main units to be installed, the Peck impact tap was located at the nose tip. On the fifth main unit it was placed at a slight angle to the longitudinal axis of the stay vane, $\frac{3}{4}$ in. from the nose tip, and on the sixth unit to be installed at a still larger angle, that is, 45 deg and $2\frac{1}{16}$ in. from the stay-vane tip. At the same time, some shift in upstream direction of the Peck low-pressure tap was also made on the latter two units.

In addition, two auxiliary piezometer openings were located at

² "Improved Type of Flow Meter," by I. A. Winter, Proc. American Society of Civil Engineers, vol. 59, part 1, 1933, pp. 565-584.

³ "Two Methods of Measuring Water to Hydraulic Turbines," Power, vol. 77, March, 1933, pp. 126-127.

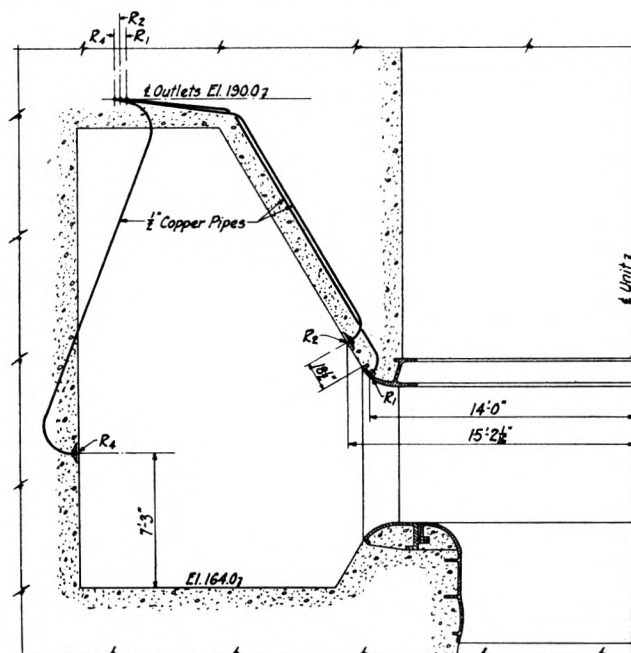


FIG. 1 WINTER-KENNEDY PIEZOMETER-SYSTEM INSTALLATION AT MAIN UNITS (Taps R_1 , R_2 , and R_4 .)

the downstream nose of one of the intake piers of each main unit for possible use should pumping with the units ever be resorted to for peak storage requirements during low flow. Both service units were provided with two piezometers of the Winter-Kennedy type. To prevent air pockets, all piping leading to the individual piezometer openings was placed with a continuous slope and copper piping was used to prevent corrosion. In the pipe tunnel beneath the generator-room floor, a piezometer board with verti-

cal glass tubes was installed at each unit where the deflections could be measured in feet of water.

After placing each unit in service, it was essential, as a first step, to determine which combination of two piezometers would prove most consistent. This was done by plotting the differential pressure of any two taps against that of any one of the other possible pairs. From Fig. 3, it may be noted that the three Winter-Kennedy taps and the Peck impact tap showed a markedly better consistency than the Peck low-pressure tap Y_2 , the latter being responsible for the erratic behavior in three of the plots. On the other five main units, the results were similar with the exception that even the Peck impact tap, located closer to the nose or at the very nose tip of the stay vane, was considerably less steady. For all main units, the Winter-Kennedy taps showed a high degree of consistency.

This result should not be interpreted as a general weakness of the Peck type of system. Investigating the origin of this erratic behavior, that is, through analysis of the results with the various Peck tap locations, as shown in Fig. 2, it was found that the cause for instability, particularly of the low-pressure tap, was rather in the design of the stay vanes than in the type of piezometer system. The Safe Harbor stay vanes are comparatively short and have a straight longitudinal axis. Since, on the one hand, the low-pressure tap was erratic in all units, irrespective of the shift upstream, and, on the other hand, the consistency and magnitude of deflection of the impact tap increased decidedly by the shift away from the nose tip, it could be concluded that the stay vanes of the speed ring were not pointed head on into the flow but at a considerable angle, causing a region of local disturbance on one side of the stay vanes, with the unstable region extending almost to the very tip of the vane. In the light of these results and, in view of the experience obtained elsewhere with piezometers of the Peck type, it would appear that a considerable improvement in stay-vane design is yet to be accomplished by lengthening, better streamlining, and curving these vanes. It is noteworthy

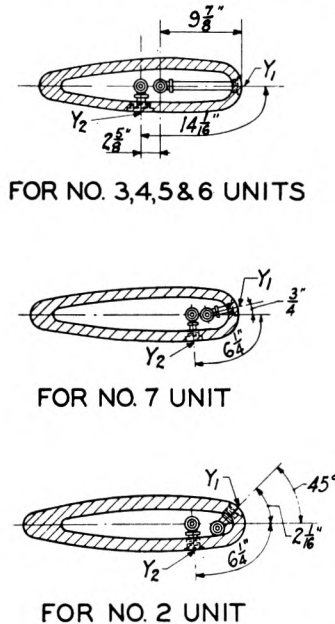


FIG. 2 LOCATION OF PECK PIEZOMETERS ON MAIN-UNIT SPEED-RING STAY VANES
(Taps Y_1 and Y_2 .)

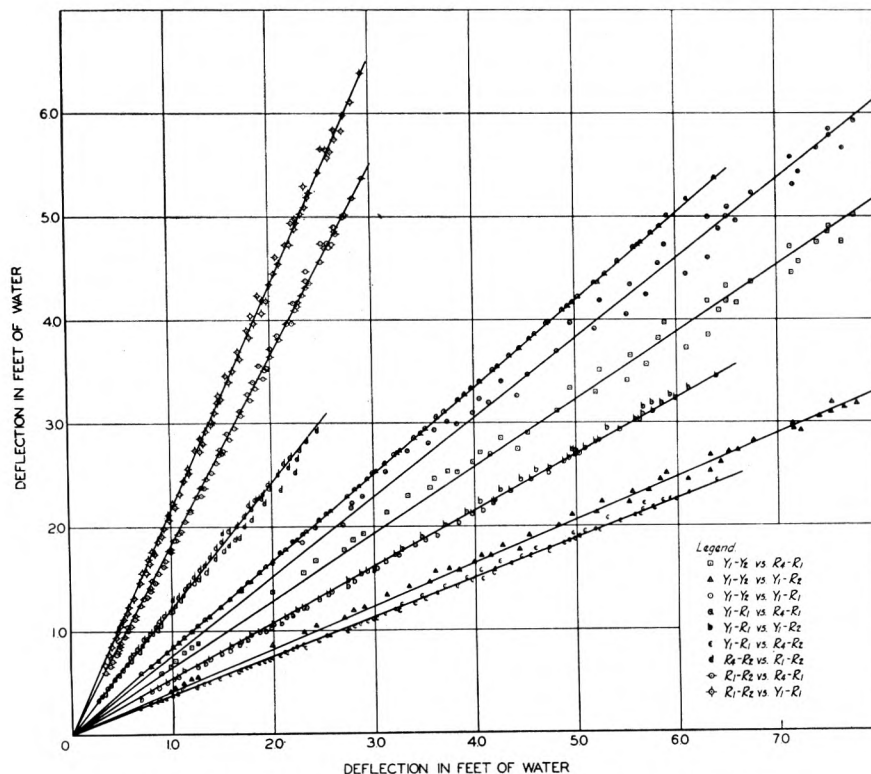
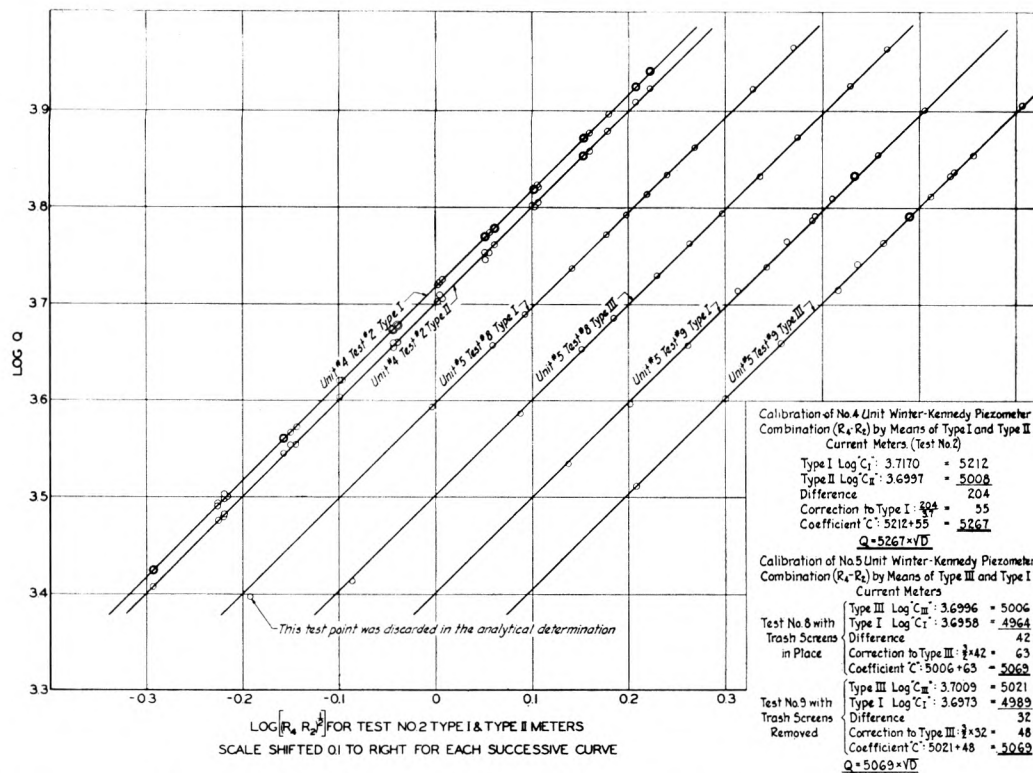


FIG. 3 ANALYSIS OF PIEZOMETER DEFLECTIONS OBTAINED AT NO. 2 UNIT

FIG. 4 CALIBRATION OF PIEZOMETER PAIR (R_4-R_2) AT NOS. 4 AND 5 UNITS BY MEANS OF TWO-TYPE CURRENT-METER METHOD

that, in more recent installations, some improvements in this direction already have been made.

While the piezometers were used initially simply as a relative index to determine the proper relation between turbine-blade and guide-vane positions under various operating heads for the Kaplan main units and served as a basis for the cam designs controlling the gate-blade relation, these piezometers were calibrated for absolute-discharge measurements in course of the acceptance tests by means of the two-type current-meter method.⁴ The results obtained for the piezometer pair (R_4-R_2) (refer to Fig. 1) of the Winter-Kennedy system of two main units are shown in Fig. 4. These curves relate piezometer deflection and discharge in accordance with the fundamental equation

$$Q = C \times D^a$$

where Q is the discharge measured in cubic feet per second and D the differential piezometer pressure in feet of water. The slope of the curves and their intercept at zero corresponding to the exponent a and coefficient C , respectively, were determined analytically, based on the method of least squares.

Of the six main units, three were tested by means of current meters.⁴ The piezometers of the other units were calibrated indirectly by assuming their peak efficiencies to be identical with those of other units of the same design and manufacture actually tested. This procedure was also followed for the two identical Francis-type service units in testing one of them by means of current meters and assuming the peak efficiencies of both to be alike.

It is recognized that such a procedure is not absolutely correct because identical units have not necessarily identical peak efficiencies. However, based on experience available, it is believed that the error thus introduced will not exceed 1 per cent for any one unit and that the average for the entire station should be

⁴ "Water Gaging for Low-Head Units of High Capacity," by J. M. Mousson, Trans. A.S.M.E., vol. 57, 1935, pp. 303-316.

even closer, because the actual efficiencies of these units might be higher or lower. The calibrations of the piezometer pair (R_4-R_2) of the Winter-Kennedy systems on the six main and the two service units are given in Table 1.

TABLE 1 CALIBRATION OF PIEZOMETER PAIR (R_4-R_2) OF WINTER-KENNEDY SYSTEMS

Main unit no.	Coefficient C	Departure from average, per cent	Calibration procedure
2	5107	-0.80	Current meters
3	5090	-1.13	Based on No. 5 unit
4	5267	+2.30	Current meters
5	5070	-1.52	Current meters
6	5185	+0.72	Based on No. 4 unit
7	5170	+0.43	Based on No. 5 unit
Average	5148		
Service unit No.			
41	330.8	-0.86	Based on No. 42 unit
42	336.6	+0.86	Current meters
Average	333.7		

Recognizing the fact that piezometers are very sensitive and greatly affected by local disturbances, due to irregularities of the water passage, as well as due to minute changes in the shape of the piezometer opening, the differences in the coefficients are relatively small.⁵ The variation in the exponent a of the equation ($Q = C \times D^a$) was also very small, varying between 0.500 \pm 0.005, so that for all practical purposes the square root was found to be of sufficient accuracy.

To guard against unexpected trouble in the future, which would render one or the other piezometer unreliable or useless, all taps were calibrated during these tests. For instance, Fig. 5 shows the calibration of No. 2 unit Peck impact tap Y_1 and Winter-Kennedy low-pressure tap (R_2) combination (Y_1-R_2).

The calibrations of the piezometers also permitted arriving at some conclusion regarding the degree of consistency and relative precision of these systems, Table 2. The consistency or average

⁵ "Piezometer Investigation," by C. M. Allen and L. J. Hooper, Trans. A.S.M.E., vol. 54, 1932, pp. 1-11.

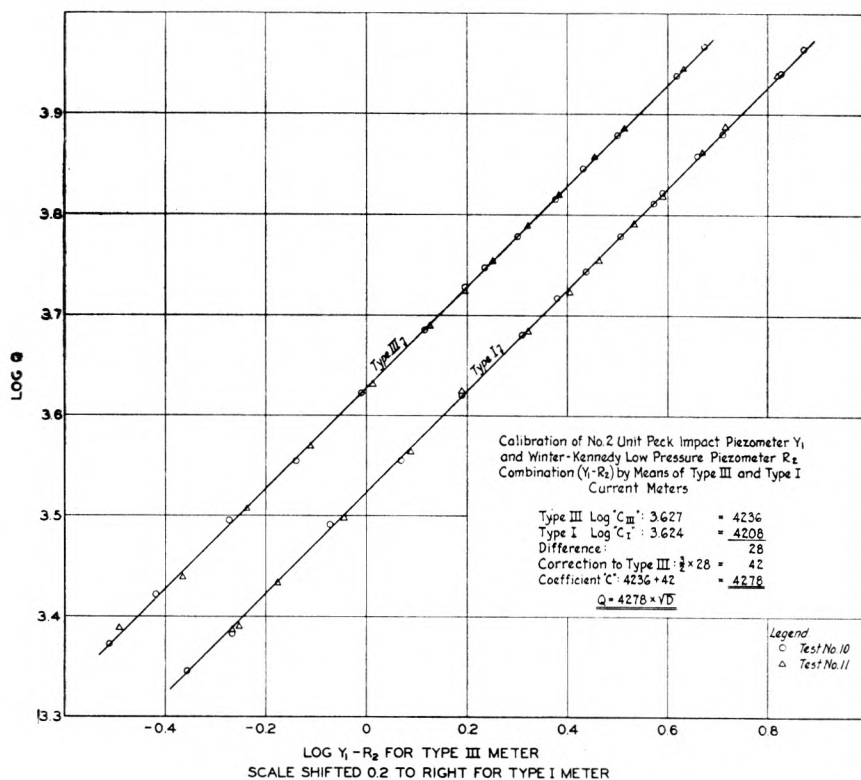
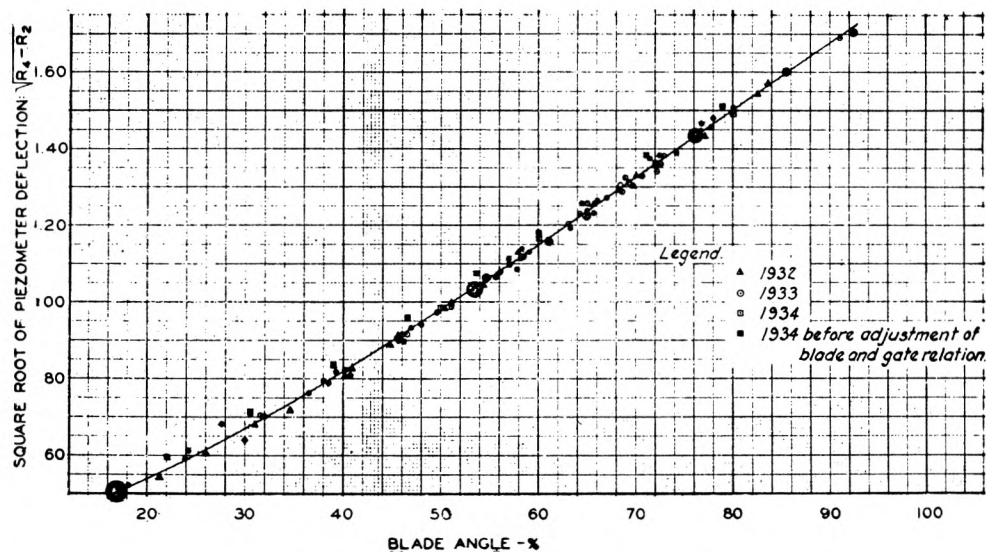
FIG. 5 CALIBRATION OF PIEZOMETER PAIR (Y_1-R_2) AT NO. 2 UNIT BY MEANS OF TWO-TYPE CURRENT-METER METHOD

FIG. 6 CHECK ON CONSISTENCY OF PIEZOMETERS USING TURBINE-RUNNER BLADE OPENING AS A PARAMETER

departure of one test point was determined analytically to ± 0.25 per cent for the measurements with the type I current meter, shown in Fig. 4. The mean departure of one measurement was ± 0.34 per cent. The mean departure of all measurements was ± 0.06 per cent and the relative precision ± 0.04 per cent. Considering that these values include not only the errors of the piezometer system but also those of the current-meter measurements, it is believed that the accuracy of the systems is fully adequate as a basis for continuous-flow measurements.

As a next step it was essential to see whether or not the piezometers would maintain their calibrations over a period of

years. The results for one unit and one pair of piezometers of the Winter-Kennedy system are shown in Fig. 6, using the runner-blade angle of the Kaplan units as a parameter. In this instance a somewhat wider dispersion of the test points as compared with that in Figs. 4 or 5, is to be expected as the measure of blade angle is not very accurate due to the inevitable lag in the blade-operating mechanism. The results indicate, however, that during the period of observation no change in calibration had taken place. Of particular interest are the test points obtained in 1934, prior to proper adjustment of the blade-gate relation which for some reason had become slightly incoordinated.

TABLE 2 DETERMINATION OF CONSISTENCY AND RELATIVE PRECISION OF PIEZOMETER PAIR (R_1 - R_2); CALIBRATION OF NO. 4 UNIT

Test Run	Turbine Discharge for Type 1 Current Meter c.f.s.	Square Root of Piezometer Deflection $\sqrt{R_1-R_2}$	Departures			d^2
			$\frac{2}{\sqrt{R_1-R_2}}$	d	Per Cent	
1	7518	1.448	5192	-20	0.38	400
2	5946	1.140	5216	+4	0.08	16
3	4752	.917	5182	-30	0.58	900
4	4768	.915	5211	-1	0.02	1
5	3687	.708	5208	-4	0.08	16
6	3735	.718	5202	-10	0.19	100
7	3182	.604	5268	+56	1.07	3136
8	3145	.605	5198	-14	0.27	196
9	3163	.608	5202	-10	0.19	100
10	5245	1.006	5214	+2	0.04	4
11	6594	1.268	5200	-12	0.23	144
12	6571	1.262	5207	-5	0.10	25
13	8729	1.664	5246	+34	0.65	1156
14	8697	1.666	5220	+8	0.15	64
15	8415	1.613	5217	+5	0.10	25
16	8401	1.612	5212	0	0.00	0
17	7875	1.510	5215	+3	0.06	9
18	7444	1.424	5228	+16	0.31	256
19	7426	1.427	5204	-8	0.15	64
20	6611	1.278	5173	-39	0.75	1521
21	6651	1.276	5212	0	0.00	0
22	5987	1.153	5193	-19	0.36	361
23	5994	1.155	5190	-22	0.42	484
24	5887	1.127	5224	+12	0.23	144
25	5858	1.127	5198	-14	0.27	196
26	5289	1.011	5231	+19	0.36	361
27	5301	1.015	5223	+11	0.21	121
28	4726	.906	5216	+4	0.08	16
29	4712	.906	5201	-11	0.21	121
30	4174	.797	5237	+25	0.48	625
31	4173	.801	5210	-2	0.04	4
32	3632	.697	5211	-1	0.02	1
33	3634	.697	5214	+2	0.04	4
34	3115	.595	5235	+23	0.44	529
35	3091	.595	5195	-17	0.33	289
36	2657	.510	5210	-2	0.04	4
37	2664	.510	5224	+12	0.23	144
Avg			=5212		Avg = -0.25	$\Sigma d^2 = 11537$

ResultsConsistency or average departure of one measurement = $\pm 0.25\%$ Mean departure of one measurement = $\pm \sqrt{\Sigma d^2 / (n-1)} = \pm \sqrt{11537/36} = \pm \sqrt{320.472}$
= $\pm 17.90 = \pm 0.34\%$ Mean departure of all measurements = $\pm \sqrt{\Sigma d^2 / n(n-1)} = \pm \sqrt{11537 / (37 \times 36)} = \pm \sqrt{8.661}$
= $\pm 2.94 = \pm 0.06\%$ Relative Precision of all measurements = $0.674 \times 0.06\% = \pm 0.040\%$

This may be regarded as one example demonstrating the degree of sensitivity of piezometers and how useful they may be to detect improper operating conditions.

FLOWMETER INVESTIGATION

During 1934 and 1935, three types of flowmeters, each employing a different principle, were investigated in detail to determine which type would meet the rigid requirements or could be further developed to a satisfactory stage. Aside from a minimum amount of maintenance desired, the chief requirements stipulated were a high degree of accuracy and sensitivity over the useful range and the possibility of totalizing the unit flow automatically for the entire station, as well as metering characteristics permitting short duration tests on each unit to determine its efficiency. The basic principles employed by these types of meters were as follows:

For the first type of meter the differential pressure of two piezometer taps served to establish flow in a system, the intake being the high-pressure tap and the exit the low-pressure tap, the rate of flow through this system varying with the differential pressure or discharge through the turbine. The meter consists of a drum about 10 in. diam and 5 in. long with the axis of the drum or cylinder in a horizontal position. A vertical partition divides the drum into two half-cylindrical chambers. This partition supports the hollow central core of the drum. If the drum

were split open its cross section would be similar to a wheel with two spokes. In the central hollow core of the drum, there is located a knife-edge bearing or a ball bearing to allow the drum to swing back and forth. The two drum chambers are interconnected through an orifice located near the lower end of the partition, that is, near the drum periphery.

By utilizing the flow through the system to displace mercury from one half-cylindrical drum chamber into the other through the orifice, there results a rotational movement of the drum around its own axis. Water displaced by the mercury in the second drum chamber is discharged through the low-pressure tap. When reaching a certain predetermined limit of tilting, a four-way cock operated by a mercury switch is turned 90 deg, changing the feed from the high-pressure tap to the other drum chamber and also connecting the low-pressure tap to the opposite chamber, thus reversing the flow of mercury and, accordingly, the direction of rotation of the drum. In continuous operation, a cyclic rotational drum movement is obtained similar to a pendulum motion; and the larger the differential pressure, the shorter the time required for each cycle. A counter, operated by the limit mercurial switches mounted on either side of the drum, records the number of drum swings and can be calibrated to serve as a flow integrator through the turbine.

The second type of meter employed the differential-piezometer pressure to lower or raise a float or dome also through displacement of mercury, the dead weight of the float being balanced by a counterweight supported by a cable fed over a pulley. By means of gears, the motion of the pulley shaft may be utilized for instantaneous-flow indication. The integration of flow is accomplished through a clock-operated disk driving a small wheel attached to the cable. The cable movement changes the position of the small wheel and places it at a certain distance from the disk center. While at a high rate of flow, the small wheel is placed close to the disk periphery and, therefore, operating under a high gear ratio, it is placed in the disk center at zero position of the meter and, consequently, does not rotate at all. The small wheel driven by the disk in turn operates an integrating counter. At the same time, a graphic record of the unit discharge can be obtained by means of a pen recording the cable movement or position on a drum making 1 revolution per 24 hr.

The third meter type employed a radically different principle. It is based on the fact that the centrifugal force exerted by a fly-ball system has the same relation to the rate of rotation that the differential pressure has to the rate of flow. The essential parts of this instrument are a tilting mercury manometer and a motor integrator carrying a flyball system, so arranged that the centrifugal force due to rotation of the integrator is opposed to the force of the tilting manometer, Fig. 7. A mercury switch operated by the beam of the tilting manometer controls the motor speed, maintaining a balance between the centrifugal force of the motor integrator and the piezometer-differential force acting upon the tilting manometer. Since the force due to the piezometer differential varies with the square of the flow being measured and the centrifugal force of the integrator also varies with the square of the integrator speed, the two square laws accordingly cancel, thus leaving a direct relation between flow and integrator speed. A counter geared to the motor integrator shows revolutions in terms of flow.

While the first type of flowmeter referred to was found to be extremely accurate even for very low differential pressures, that is, low turbine discharges, its main disadvantage lay in necessitating a continuous flow through the piezometer piping system. For the measurement of gas, steam, filtered water or any refined fluid, there would be no danger from plugging up the piping, but with silt-laden river water, such as carried by the Susquehanna River, there was great danger of rendering the entire piezometer piping

system useless, even with frequent flushing by compressed air or filtered water. At the same time, this apparatus did not lend itself particularly to totalizing, because a like periodicity of the drum motion on different units would not correspond to equal unit discharges, due to the difference in the piezometer calibrations. Theoretically, it could be compensated for by introducing different gear ratios for the individual counters or by changing the size or location of the orifice connecting the two drum cham-

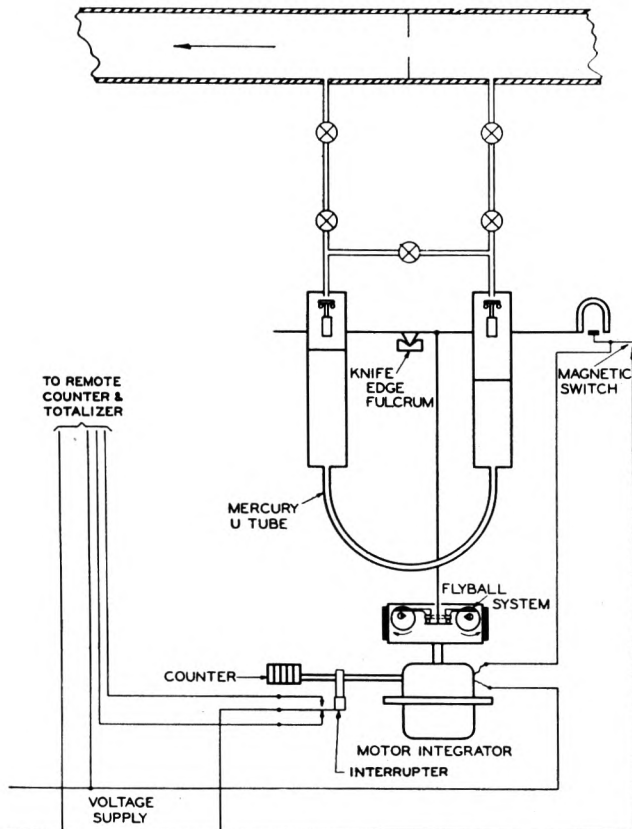


FIG. 7 DIAGRAMMATIC SKETCH OF TYPE 3 FLOWMETER
(By courtesy of the Leeds & Northrup Company.)

bers or even through adjusting the amplitude of the cyclic rotational drum movement. In view of the various disadvantages and complications, this type of meter, though accurate, could not be given any further consideration.

Two flowmeters of the second type were purchased and installed temporarily on separate units in the pipe tunnel beneath the generator-room floor. Prior to shipment, these meters were calibrated by the manufacturer for the respective piezometer systems. As a first step, hourly readings of the meters were compared with analytically determined unit discharges based on output. As expected, the flowmeters showed consistently larger unit discharges, the discrepancy being more the greater the fluctuation in loads carried by the units. With the units operating on hand control and blocked to generate at a constant output, there was close agreement between metered and analytical discharges. These results may be attributed to the concave shape of the unit-efficiency curves.

Next, these flowmeters were used to make turbine-efficiency tests of 5- and 10-min duration, the flowmeters being read every 15 sec and the watt-hour-meter-disk revolutions and the time in seconds being recorded by a chronograph. On each unit, the test points thus obtained spread considerably over a band about 3 per cent in terms of efficiency. This discovery led to analysis

of the instrument errors by making standard water-column tests. Three major sources of errors were revealed. A first error was traced to the eccentricity of the integrating disk. This error was not constant but had a periodic sinusoidal characteristic completing the cycle in $\frac{1}{2}$ hr, corresponding to the time required for 1 revolution of the disk. While for one of the flowmeters the amplitude of this sinusoidal-error curve varied between +0.63 and -0.38 per cent, the other flowmeter showed disk-error variations between -0.82 and +1.39 per cent. The second error, which could not be controlled, was the variable frequency of the station-service system from which the clock driving the integrating disk obtained its power supply. Since the frequency varied about 1 per cent, it was sufficient to make the disk error inconsistent with time, by causing a phase shift in the disk-error curves.

The third source of error was due to the lap of the counter gears and the weight of the rotating countersweep hand, its weight tending to accelerate the motion in the downstroke and retard it in the upward swing. This phenomenon superimposed

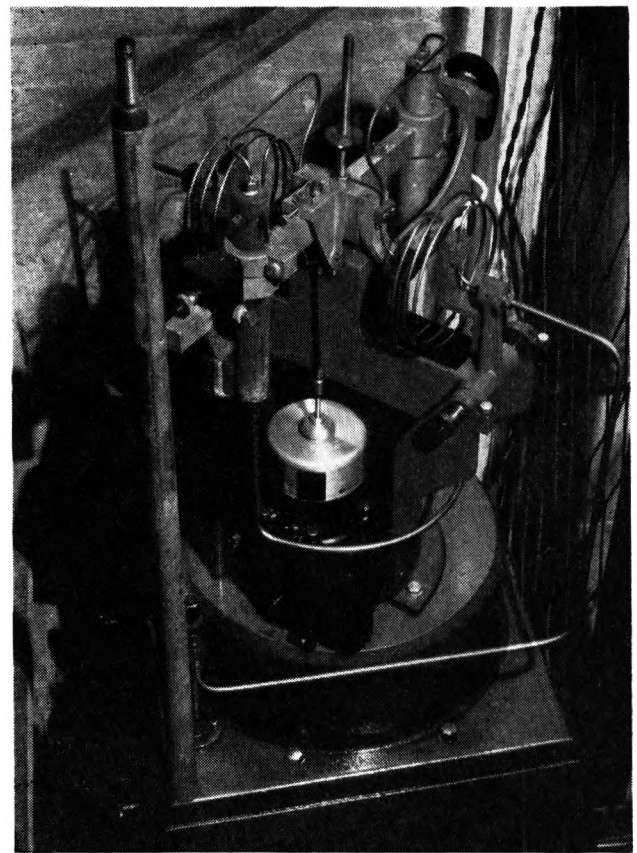


FIG. 8 TEMPORARY INSTALLATION OF EXPERIMENTAL TYPE 3 FLOWMETER AT NO. 5 UNIT

another error of periodic characteristic upon the first error referred to. To make matters more complicated, the frequency of the second periodic error was not constant but varied with the discharge, being a function of the speed of the countersweep hand and, therefore, decreasing with increasing discharge.

Although results of short-duration efficiency tests, using compensating measures for the errors referred to, gave greatly improved results, it was realized that such procedures were too complicated to be adopted as a routine measure on all units, because the effort involved, in analyzing instrument errors for meters to

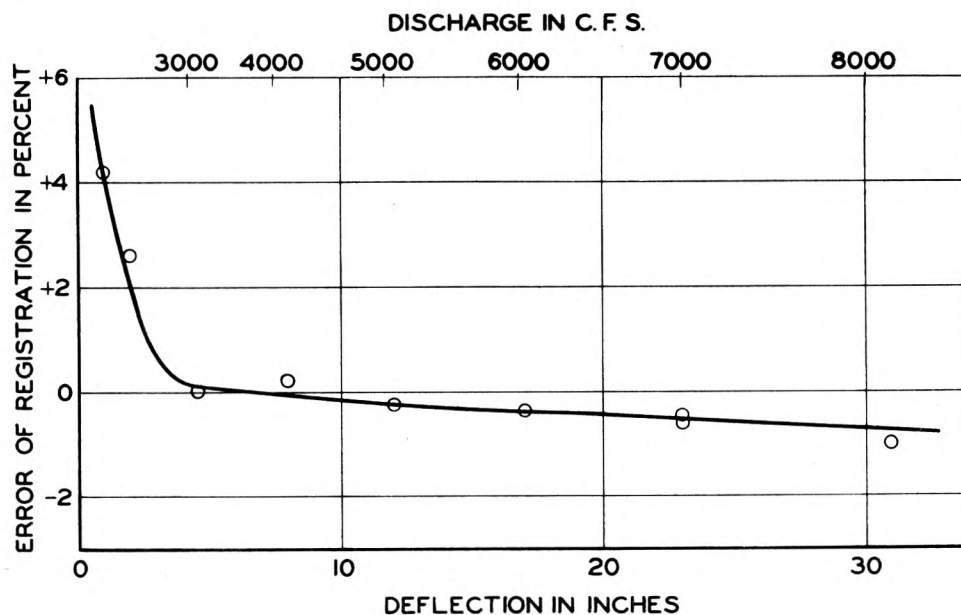


FIG. 9 ERROR CURVE OF EXPERIMENTAL TYPE 3 FLOWMETER

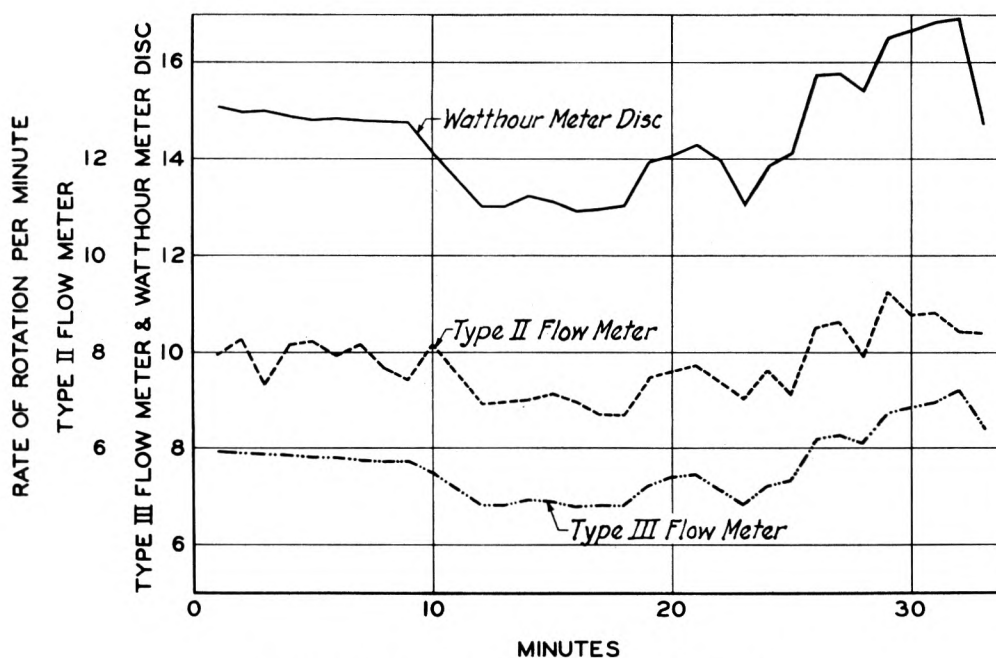


FIG. 10 SENSITIVITIES OF TYPES 2 AND 3 FLOWMETERS

be installed on all units and the use of difficult and complicated compensating procedures for each meter, was far too great in comparison with the accuracy of the results obtained.

An experimental flowmeter of the third type was installed temporarily on one of the units, Fig. 8, and operated in parallel with one of the meters of the second type. Hourly readings on both meters were compared with each other as well as with discharge computations based on power output. The two flowmeters agreed within 0.5 per cent in average, the third type of meter being closer to the analytically determined discharge.

Short-duration efficiency tests on the third type of meter showed a very small spread of test points and a very good agreement with the results of the turbine acceptance tests. The execution of these tests could be simplified considerably by being

able to record the flowmeter countershaft revolutions, Fig. 7, by means of electrical impulses on the chart of a recorder simultaneously with the revolutions of the watthour-meter disk and second impulses. The watthour-meter-disk revolutions were obtained by means of a photoelectric-cell arrangement, a small electric bulb being placed on one side of the disk and the photoelectric cell on the opposite side. The beam of light causing the impulses fell through the balancing hole in the watthour-meter disk. The second impulses were obtained also by means of a photoelectric-cell arrangement mounted on the master clock for frequency control of the system, the beam of light being cut by the pendulum.

Next, the meter errors of the third type of flowmeter were analyzed by means of the standard water-column tests. As may

be seen, the error curve as shown in Fig. 9 had the typical shape of a rotational integrating device. While the test points were rather consistent, nevertheless it was concluded that further improvement of the meter should be carried out to flatten and lengthen the horizontal leg of the error curve and improve its accuracy to such a degree that even analytical compensating measures would not be required for short-duration efficiency measurements on the turbines.

Additional tests were carried out to determine the responsive-

ness and sensitivity of the second and third types of flowmeters with varying load on the generating unit. The results, shown in Fig. 10, demonstrate the consistency of the third type of flowmeter, as it follows the watt-hour-disk-revolution indications consistently in contrast to those of the second type.

In view of the fact that the totalizing with the third type of meter was a simple electrical problem and well-established principle, it was decided to use the third type of flowmeter for the Safe Harbor installation, provided satisfactory improvements were made by the manufacturer in the error characteristics.

2—FLOWMETER EQUIPMENT INSTALLED AT SAFE HARBOR

During 1936 and 1937, various studies were made on remote unit-discharge-totalizing equipment. The manufacturer's attention was drawn also to the possibility of using this type of equipment as part of unit- and station-efficiency indicating-and-recording apparatus. By 1938, the plans for such an installation had crystallized to a point where it was felt safe to proceed with the installation of the flowmeter equipment for all units, as well as the flow-totalizing apparatus for the entire station.

The flowmeters selected were installed in cabinets originally provided on the generator-room floor, located adjacent to and forming an integral part of the gage boards of each unit, Fig. 11. This installation comprised eight flowmeters, one for each of the six main units and one each for the two service units. On all units the flowmeters were connected to the Winter-Kennedy piezometer pair (R_1 - R_2) and calibrated, based on the data given in Table 1.

It should be noted that the error characteristics of these meters had been materially improved, so that no correction of any sort had to be applied over the entire range of turbine discharge actually used. The improvement in the error characteristics can best be realized by comparing the check calibrations of three flowmeters after installation, Fig. 12, with the results obtained with the experimental flowmeter in 1934, which is shown in Fig. 9.

Another desirable advantage of these meters is that the checking or recalibration is greatly simplified by calibrated weights to be hung on one arm of the tilting mercury manometer, thus eliminating the use of standard water columns. As demonstrated by the data plotted in Fig. 12, the results obtained by each method are, for all practical purposes, identical.

The power supply for the flowmeter motor integrators was obtained from the 120-v 60-cycle station-service system at outlets

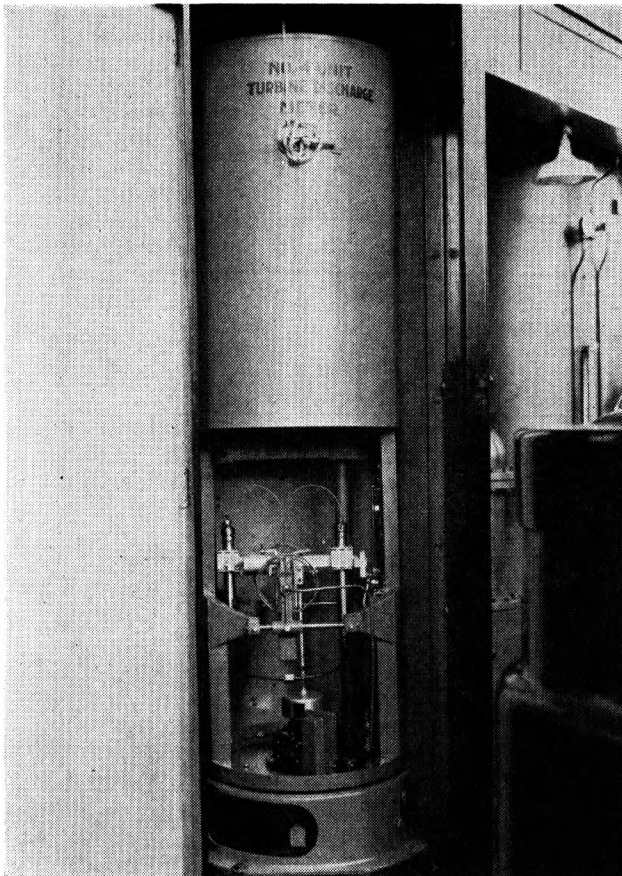


FIG. 11 TYPICAL FLOWMETER INSTALLATION

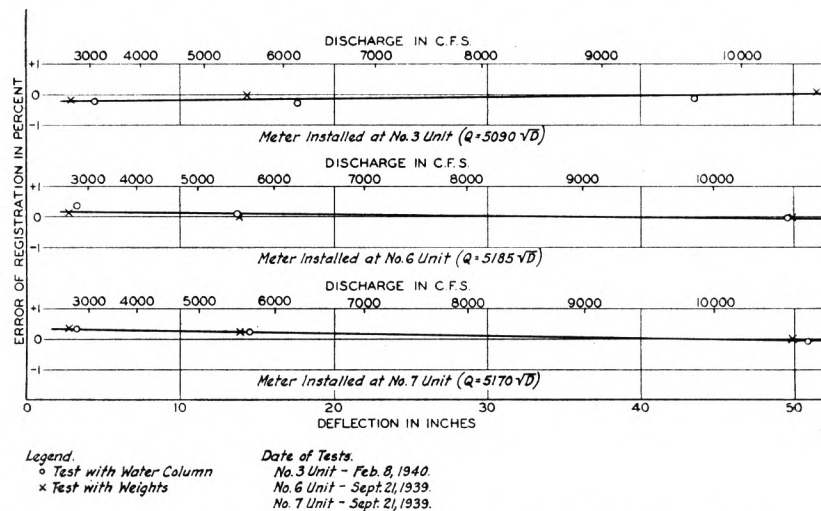


FIG. 12 CHECK CALIBRATIONS OF THREE TYPE 3 FLOWMETERS AFTER INSTALLATION

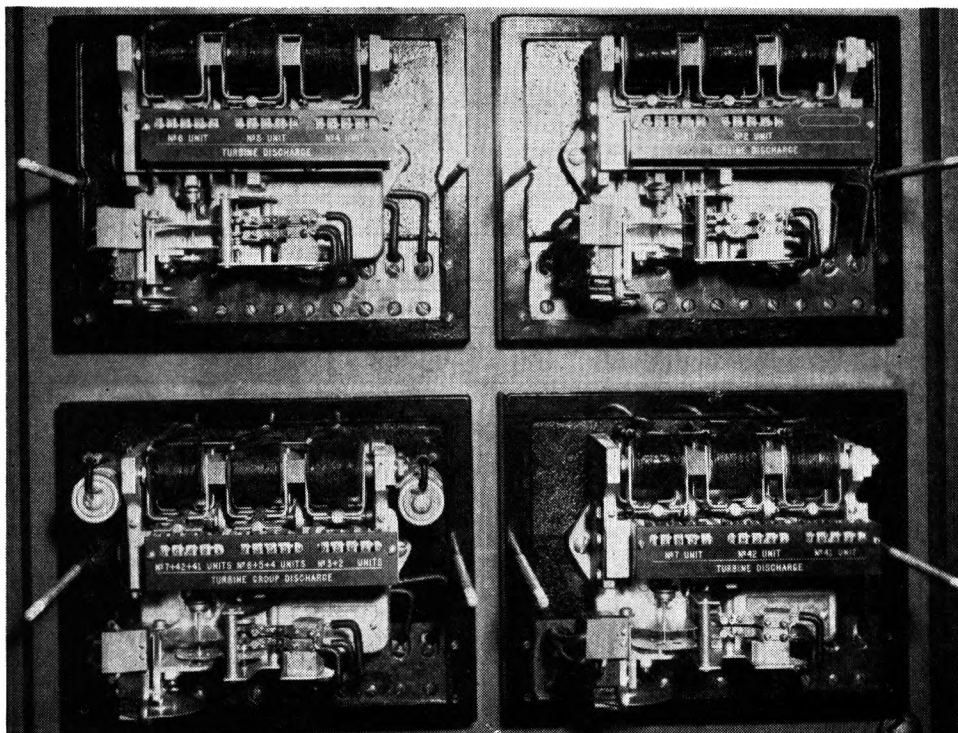


FIG. 13 DISCHARGE TOTALIZING RELAYS

available at each unit gage board. The totalizing apparatus was installed on a panel of the relay board in the control room, Fig. 13. Its principal parts consist of four impulse totalizing relays. Three of these serve as unit-discharge totalizers for a group of three turbines each and one as master totalizer for the entire sta-

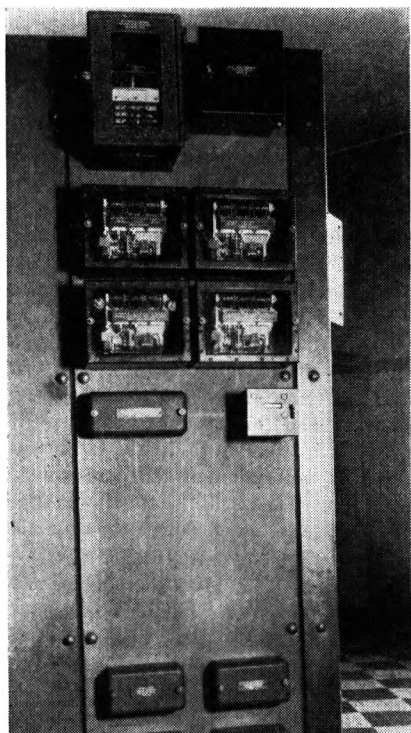


FIG. 14 TOTALIZING RELAYS AND STATION TOTAL DISCHARGE COUNTER INSTALLED ON RELAY BOARD IN CONTROL ROOM

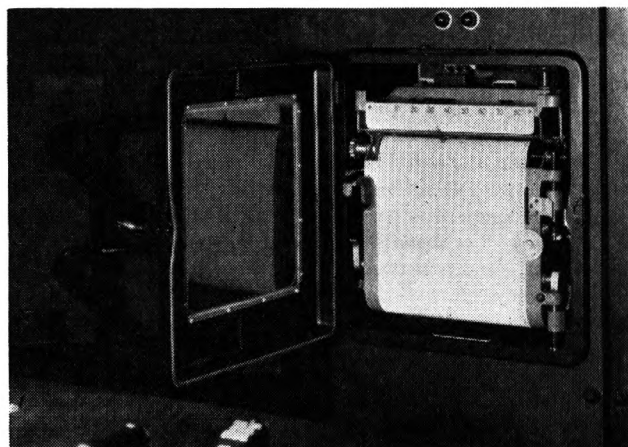


FIG. 15 STATION TOTAL DISCHARGE INDICATOR AND RECORDER INSTALLED ON INSTRUMENT BOARD IN CONTROL ROOM

tion, giving the sum total of the three unit totalizing relays. The spare position on the first totalizing relay will be used for the flowmeter at No. 1 unit, now being installed. While the input-output ratio of the unit totalizing relays is 5:3, the master totalizer has a ratio of 3:1. Since each impulse sent out by the interrupter on the countershaft of the individual flowmeters represents 20,000 cu ft, each impulse received by the station total discharge counter from the master totalizing relay corresponds to 100,000 cu ft. The station-total counter is mounted below the totalizing relays on the same panel, Fig. 14. Individual unit discharges can be read on the individual impulse counters of the unit totalizing relays. Mechanical counters were provided on all flowmeters in order to facilitate the checking of impulse transmission and relay operation, as well as for rechecking the calibrations of the flowmeters themselves.

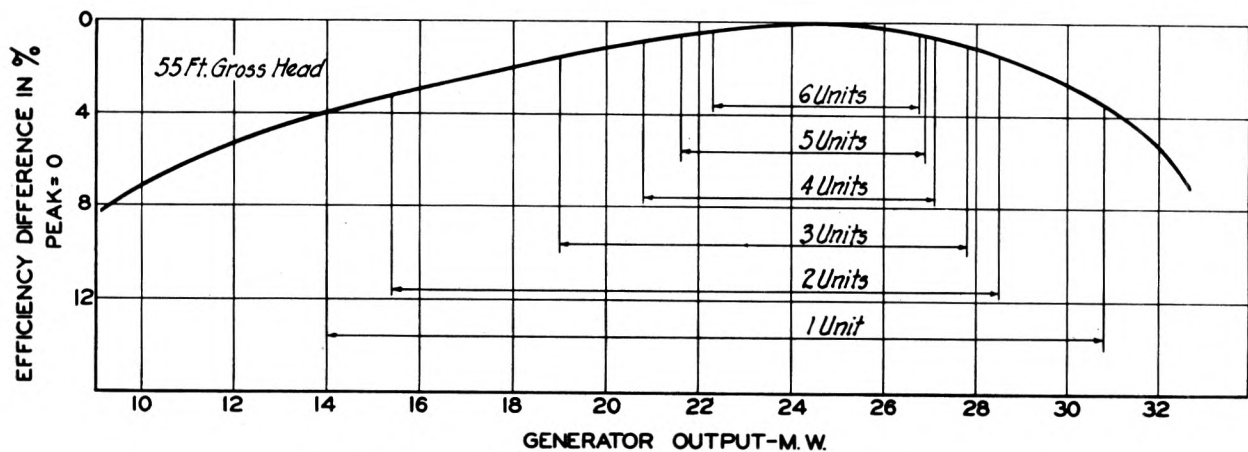


FIG. 16 TYPICAL LOADING SCHEDULE FOR SAFE HARBOR MAIN UNITS

A total station discharge graphic recorder and discharge indicator, combined in one instrument, was installed on the instrument panel located opposite the totalizing-relay panel, Fig. 15. The upper range limit of the graphic recorder and the indicator was chosen as 80,000 cfs, representing the approximate maximum station draft of the Safe Harbor development for a number of years in the future.

3—BENEFITS OBTAINED THROUGH FLOWMETER INSTALLATION

During 1939, various investigations were made based on the data obtained by the flowmeter installation. The operators were required to read the individual unit discharges and the station-total draft every hour on the hour, together with the unit and station integrating watt-hour meters, as well as forebay and tailwater indications. The operators, however, were still charged with computing the individual unit and total-station drafts based on power output, as had been standard practice. It was felt that a long-term comparison was essential to obtain the proper basis for continuous-flow records at Safe Harbor, the transition period furnishing the ratio between computed and automatically recorded station drafts under the various seasonal loading schedules. Once sufficient data have been accumulated, it is expected that the operators will be relieved altogether from computing the discharge based on output.

The data obtained by the operators were also used to investigate unit and station operating efficiencies. An investigation of this kind was all the more essential, as during approximately 290 days of the year the available river flow at Safe Harbor is less than the station draft required with all six main and two service units installed operating at maximum capacity. After placing the seventh main unit in service, which is now under construction, the corresponding period will increase to 305 days. It was interesting to note that the availability of an input yardstick had a decidedly stimulating effect on the operating personnel. While, during the first month of flowmeter operation, that is, January, 1939, the ratio of actual loss in generation to expected loss was greater than unity on all but 8 days, this ratio did not exceed unity during 18 days in May, 1939, under similar river-flow conditions and loading schedules. It has been estimated that an improvement of this magnitude is responsible for an increase in generation of at least 0.3 per cent, or approximately 1,700,000 kw-hr per year with the present installation of six main units and 1,900,000 kw-hr per year with the seventh main unit placed in service, so that the flowmeter installation will pay for itself in a very short time.

With a close continuous check on unit operating efficiencies available, it was also possible to keep the losses due to trash on the intake screens appreciably below those which must have been prevailing during previous years. Prior to the installation of the flowmeters, the screen losses were determined from time to time by measuring the screen head loss. Now, as soon as any of the units show a drop in operating efficiency, as indicated by the hourly readings, the screen losses are determined independently. After cleaning the racks, the operating efficiency invariably increases to the expected level. Although it is difficult to estimate the increase in station economy due to this means of obtaining an earlier indication of the loss in efficiency due to plugging up of the screens, nevertheless, it is believed that the benefits thus derived are substantial.

Since the availability of the particular type of flowmeters permitted short-duration turbine-efficiency tests to be carried out by one man, therefore justifying itself as a routine measure, it was also possible to investigate in detail the efficiency characteristics of each main turbine over the entire range of operating heads. Such a procedure was particularly desirable as these turbines are of the Kaplan type, requiring an adjustment of the cam controlling the gate-blade relation for the various operating heads, the operators being required to change to a new cam setting after each 1 ft of change in head. While these compensating devices on all main units were originally designed and calibrated, based on a minimum of information due to the costly testing procedures even with the use of the index method with piezometers, the flowmeters available made it possible to check and recalibrate these compensating devices with a large amount of detailed information obtained with a minimum of effort. The scope of the work involved can best be realized by mentioning that, although five of the six main units were of identical design, nevertheless, the characteristic of each unit was found to be sufficiently different from the others to warrant individual cams, consequently requiring individual calibration of the cam-adjustment device for variation in head. The results of this investigation reflected favorably upon the operating efficiencies of the individual units and the station as a whole.

As a next step, a detailed study was undertaken to determine the magnitude and duration of avoidable inefficient operation in percentage of total operating time. From the ideal loading schedule for the main units, as shown in Fig. 16, and valid for a gross head of 55 ft, it is apparent that the band of permissible load variations of each unit decreases with increasing number of units on the line. With capacity requirements above the most efficient station operating range with all available units operating,

all individual unit loadings are increased by equal amounts up to the point of maximum capacity.

By analyzing the chart of the total station discharge recorder, Fig. 15, in the light of the operator's log, it was noted that considerable periods elapsed between the time of placing units on or off the line and the ideal loading schedule. The losses thus sustained, though by no means excessive, when compared with some other stations were nevertheless appreciable, amounting to about 10 per cent of the total operating time on the average.^{6,7} It was realized that there was considerable room for improvement provided proper means were available for giving instantaneous warning when reapportioning of load to individual units is required. It is obvious that in this connection some thought was again given to efficiency-indicating-and-recording apparatus, but another and far simpler and less expensive solution was discovered.

4—INSTALLATION OF LOAD-LIMIT LIGHTS

The characteristics of the Kaplan-type main turbines installed at Safe Harbor are such that the most efficient discharge range of these units is, for all practical purposes, independent of the head if the loading schedules, valid for each head similar to that in Fig. 16, are adhered to. Thus the discharge, with one unit operating within the permissible load range, varies between 4000 and 8200 cfs irrespective of the head, and the discharge ranges

TABLE 3 DISCHARGE LIMIT SETTING FOR LOAD LIMIT LIGHTS

Discharge range, no.	Discharge-range setting, cfs	Main units to be operated, no.
1	0-4000	0
2	4000-8200	1
3	8200-14900	2
4	14900-21400	3
5	21400-27500	4
6	27500-33900	5
7	33900-40800	6
8	40800-48000	7

with any given number of units operating are also constant, i.e., independent of the head for all practical purposes. In view of these characteristics and taking proper account of station-service unit draft requirements, it was possible to provide for an automatic and instantaneous load-limit indicating apparatus as an integral part of the total station discharge indicator and recorder, shown in Fig. 15.

Essentially, this device consists of a contact-making cam arrangement controlling two warning lights, one located on the operator's desk and the other on the instrument board above the station total discharge recorder. For each load range between two discharge limits, Table 3, there is available one contact-making cam assembly independently adjustable as to what part of the total-discharge range it will control. A control switch is provided with one position for each discharge range, that is, number of units to be operated, connected so as to keep the light extinguished when set for the number of units to be in operation for best efficiency, as long as the discharge is in the corresponding range. If the discharge crosses the limits of this range, the lights will be lighted from the contact assembly of the adjacent range either until the switch has been reset to the number of units, corresponding to this new range, or the discharge has returned within the range. With the control switch being kept set correctly, that is, corresponding to the number of units in operation for best efficiency in each discharge range, the illumination of the lights will indicate inefficient operation.

It may be noted that eight discharge ranges have been provided, the reason being that the seventh main unit is now being installed and that an indication is also desirable when the station as a whole, with all seven main units operating, has reached the upper limit of the range of most efficient operation.

In addition, the scope of the total discharge station indicator and recording instrument will be increased by means of adding a load-operating-range scale, each division of this scale corresponding to the permissible range of discharge for a certain number of units in operation as shown in Table 3. By means of this improvement, it will be possible to observe at a glance how many units should be in operation at any time. When reaching a load limit as indicated by the warning lights and observing the shape of the discharge curve plotted by the station-discharge recorder, it also will be immediately apparent whether an upper or lower limit has been reached, requiring one unit to be put on or off the line, respectively.

To keep a definite record of inefficient operation, the station total discharge recorder is also to be equipped with an additional pen element operating simultaneously with the load-limit lights. This added provision will also enable the operators to ascertain the duration of the period of inefficient operation prior to noticing the lighted load-limit lamps, so that the allowable 10-min interval of borderline operation is not exceeded. Some inefficient operating time is necessarily unavoidable and, for the present and some time past, we have felt that a 10-min period of allowable inefficient operating time is reasonable.

The installation of this load-limit light apparatus is now in progress and it is expected that, due to its availability, avoidable inefficient operation will be reduced to a negligible amount, resulting in an additional and substantial increase in operating efficiency and station output.

Discussion

M. M. BORDEN.⁸ The type 2 flowmeter referred to was not constructed with certain precise operations, involving gear-centering and tooth-spacing in particular, which are applied to instruments the totalized flows of which are to be read at intervals of a few minutes rather than several times a day.

A case in point is taken from the record of one of several such meters, which were furnished for an electric power station. In this instance the maximum errors of the totalizer when read at 5-min intervals varied from -0.8 to $+0.9$ per cent with an average for a 90-min period of $+0.16$ per cent.

For 10-min intervals between readings, such point errors varied from $+0.65$ to -0.65 per cent and the 90-min average was $+0.16$ per cent.

For 30-min intervals between readings, the point errors were from a maximum of $+0.3$ to -0.1 per cent, with a 90-min average of $+0.1$ per cent.

The errors were determined by comparing the readings of the fast-moving hand of the totalizer with its 4-in. graduated circle with the record of the water weighed in the laboratory tanks.

The type 2 instrument permitted comparison of the instantaneous rate of water flow with the corresponding instantaneous indications of the electrical output and of the head on the wheels.

The water-flow rate indication of this meter is made without the use of gearing and may be read by a pointer moving in front of an equally spaced flow scale of whatever radius is required.

While the W - K relationship appears to have a normal flow of 0.5 for n , the operating principle of the type 2 meter allows it to be furnished with a uniformly spaced flow scale for any value of n which the particular field rating might necessitate.

⁸ Chief Engineer, Simplex Valve & Meter Company, Philadelphia, Pa. Mem. A.S.M.E.

⁶ "How We Raise Hydro Efficiencies," by E. B. Strowger, *Electrical World*, vol. 103, April 14, 1934, pp. 535-538.

⁷ "Waterwheel Testing and Operating Records of Plant Discharges," Proceedings National Electric Light Association, vol. 85, 1928, pp. 872-904.

E. S. BRISTOL.⁹ This paper presents an interesting review of the steps taken in a persistent investigation that finally resulted in a flow-measuring installation of a rather unusual nature. It is of interest also to note how the various obstacles were overcome through careful study and how the information yielded by the final metering system was analyzed to obtain improved station performance.

Additional information with respect to the type 3 flowmeter described by the author, will make more apparent the characteristics contributing to the degree of accuracy reported. Referring to Fig. 7 of the paper, it is seen that the flowmeter is a force balance in which a force dependent upon piezometer pressure difference is opposed to centrifugal force from a rotating-flyball system. The meter is a relay-type mechanism, in which the balance arm functions only as a detector to regulate electric-power supply to the integrator motor. A knife-edge support is provided for the balance arm which is not required to operate any indicating, recording, or integrating elements, but which merely functions to actuate a magnetically operated mercury switch. The alternate closing and opening of the mercury switch results in an on-off control of the integrator motor, such that its speed oscillates slightly above and below the required average value for any particular pressure differential. The tilting manometer thus has a continuous rocking action, similar to that of many speed governors, which reduces to a minimum any tendency of the mercury to stick to the manometer tubes as well as any frictional effects.

The flyball system, actuated by the integrator motor, is of the neutral type, such that force transmitted to the manometer arm is independent of the flyball angular position over the working range. This characteristic avoids change in calibration when the manometer arm assumes slightly different average positions, as required to change the on-off time cycle of the mercury switch in maintaining required motor speed, despite variations in voltage, frequency, etc.

The integrating element of this type of meter inherently possesses the same accuracy as the meter itself, since direct coupling of the integrator to the variable-speed motor, driving the flyball system, avoids the introduction of any intermediate errors.

Separate means of adjustment are provided for calibrating the high and low ends of the meter range, thus making it possible to match closely the characteristics of the primary-flow element, such as the turbine-scroll piezometers employed at Safe Harbor, or the venturi tube, flow nozzle, or thin-plate orifice more commonly used. The high-range adjustment consists of a threaded rod for changing the point of attachment of the vertical flyball link to the horizontal manometer arm. The low-range adjustment consists of a moving balance weight on the manometer arm. By means of these adjustments, the meter calibration can be readily changed in the field to suit an experimentally determined coefficient of the primary element, in applications where facilities are available for checking the latter in its service location. Figs. 9 and 12 of the paper indicate the nature of the variations which can be made in the meter calibration. The experimental meter of Fig. 9 was slow at the higher flows, so that the manometer balance weight was offset in the increase direction to improve the over-all relation. The three calibration curves of production meters, in Fig. 12, show much improved settings at high flows, with both high and low deviations at low flow, depending upon the particular low-range-adjustment setting. As pointed out by the author, once the relation between water-column readings and check-weight readings has been determined, the latter can be used in routine accuracy checks with resultant saving in maintenance time.

⁹ Engineering Department, Leeds & Northrup Company, Philadelphia, Pa. Mem. A.S.M.E.

The relay equipment for totalizing station-water flow is of the standard impulse type employed for electrical-demand metering, with minor modifications to suit the high rate of operation required. Flow is totalized every 2 min, so that a high rate of impulses per minute is necessary in order to obtain reasonably close setting of the totalizing recorder. The pen of this recorder moves at the expiration of each 2-min interval to a position corresponding to average rate of station-water flow during that interval. The totalizing action employs positive forward and return electrical impulses, with corresponding forward and return solenoids on the totalizing relays. As a result, no false counts occur if a transmitting contact chatters and produces more than one impulse in the same direction. After each forward impulse, the associated return impulse must go through to reset the receiving element, before a successive forward impulse can be of any effect.

The author refers to the possible use of the flowmeter equipment as a component of efficiency-measuring equipment for individual generating units or for the complete station. To obtain an indication or record of efficiency, elements must be added which will properly combine effects representative of electrical output and hydraulic head with the water-flow measurement and provide an ultimate indication of the ratio of electrical output to the product of water flow multiplied by head. These operations can be performed electrically, using an emf from a thermal converter or torque balance to represent electrical load, an emf from a slide-wire, positioned in accordance with rate of flow, and an emf proportional to head, as derived from float-actuated slide-wires at the forebay and tailrace. By applying these emf values to suitable potentiometer recording equipment, a continuous record can be obtained of the efficiency of an individual unit or of the entire station.

In closing, it may be mentioned that the type 3 flowmeter is not restricted to hydraulic applications, but is also employed in steam-flow service.

F. NAGLER.¹⁰ The water-power industry has been all too slow in analyzing its own performance. Its system seems to be much less exact than that of a flour mill, a country grocery store, or a gold mine. All too frequently, however, because of the difficulty of sampling the ore, mining operations are tabulated on the basis of adding the bullion produced to the assumed or measured gold content in the tailings, that sum being reported as the "head." Water-power management has not been so greatly different in the conduct of its own affairs.

The ideal state would be to charge to the plant the flow in the river and credit to the plant the kilowatthours produced. The apparatus methods described in the paper are, apparently, sufficiently directed to that very end.

Is it not inevitable that any piezometer located on the nose of the vane will be more erratic, under variable-flow conditions, than one located on a surface where the flow is directed? This does not refer so much to the variation of flow from the operation of the guide vanes, as to variations resulting from influences further upstream. Typical sources would be the condition of the racks but, more particularly, the condition of operation of adjacent units. In any event, careful observation of the indicated results should permit attention to be called quickly to any abnormal flow condition which might be harmful.

Apparently, Fig. 16 of the paper tells quite a story, not so much for a plant containing six units, but particularly for plants which contain more. Flatness of the efficiency-gate-opening curve naturally plays less and less part in efficient operation of plants as the number of units increases. Should this be applied still further to the regulation of units, a series of curves would

¹⁰ Chief Engineer, Canadian Allis-Chalmers, Ltd., Toronto, Canada. Life Member A.S.M.E.

result very much as shown in Fig. 7 of a paper¹¹ by the writer on speed regulation.

J. F. ROBERTS.¹² This paper should be of great interest to engineers who have tried to keep accurate discharge records at hydroelectric plants. Apparently, the author's organization has been successful in obtaining the cooperation of the operating engineers. In his earlier experience the writer frequently encountered opposition or at least lack of interest in this regard. Since turbine flowmeters invariably show a greater discharge, as compared with computed discharges based on kilowatt-hour output, the operators sometimes preferred the latter method, as it gave them credit for a higher operating efficiency than actually existed.

The desirability of turbine flowmeters is now universally recognized by operators as essential in large modern hydroelectric plants. How many modern steam plants are built at the present time without accurate coal, feedwater, and steam flowmeters? Electrical engineers would not think of omitting both integrating and recording watt-hour meters, yet some of these same engineers formerly belittled the use of turbine-discharge flowmeters, preferring to rely on the unit-performance curves made up when the units are new and under ideal test conditions.

The Tennessee Valley Authority has had excellent results with the Winter-Kennedy type of taps shown in the author's Fig. 1. One set of these taps was calibrated on a 16-in. test model of a 45,000-hp 48-ft head, fixed-blade propeller turbine, obtaining the following calibration: $Q = 6562 D^{0.481}$ where D is the deflection in feet of water, the quantity of water being measured by a weir.

Gibson tests on similar taps on a 66,000-hp 165-ft head Francis turbine gave the following equation for two similar units:

$$\begin{aligned} \text{Unit 1 } \left\{ \begin{array}{ll} Q = 1693.4 D^{0.5026} & \text{for } R_6 - R_3 \\ Q = 1367.7 D^{0.5087} & \text{for } R_6 - R_1 \end{array} \right. \\ \text{Unit 2 } \left\{ \begin{array}{ll} Q = 1659.3 D^{0.5103} & \text{for } R_6 - R_3 \\ Q = 1325.2 D^{0.5088} & \text{for } R_6 - R_1 \end{array} \right. \end{aligned}$$

where D is the deflection in inches of mercury.

J. W. SCOVILLE.¹³ The stay vanes of a speed ring are a necessary evil, and their angle and shape have been objects of considerable investigation in so far as turbine efficiency is affected. The angle is necessarily a compromise, since a turbine has to operate at any gate opening. The angle and shape are such that the best efficiency is not reduced nor maximum output affected adversely. Necessarily no consideration has been given to the effect on the Peck piezometers. The writer has noticed that the coefficient for the Peck taps varies with gate opening on Kaplan turbines in several plants which have been tested. This fact does not necessarily preclude their use for index testing in connection with the determination in the field of the proper blade-gate relationship of a Kaplan turbine. The Winter-Kennedy system is equally suitable for this purpose.

The author mentions but does not stress the fact that the piezometers were used for such index testing. It is possible by such methods to obtain the correct blade-gate relationship of a Kaplan turbine without going to the expense of a water-measurement test.

As the author points out, if the deflection between Peck or

Winter-Kennedy taps is measured, Q is equal to $C \times D^a$ where a is practically 0.5. If a test is made at various blade angles at several gate openings, during which the head, kilowatt output, and deflection are measured, curves can be plotted as shown in Fig. 17 of this discussion.

The ordinates $KW\sqrt{D}$ are proportional to the unit efficiency if plotted for a constant head. In effect, such a curve is an over-

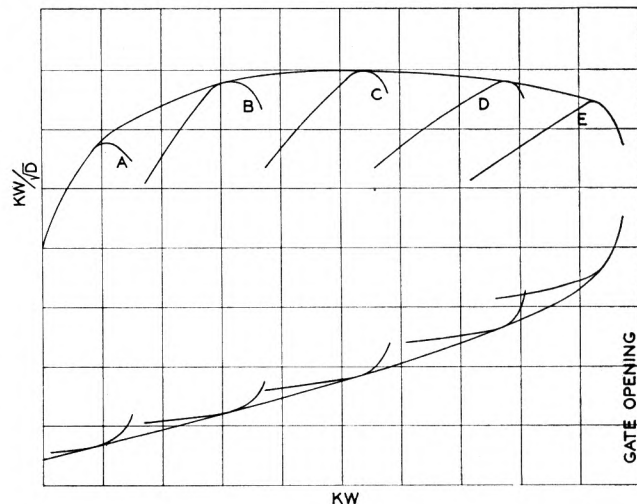


FIG. 17 CURVES SHOWING RELATION BETWEEN HEAD, KILOWATT OUTPUT, AND DEFLECTION, RESULTING FROM TESTS MADE AT VARIOUS BLADE ANGLES AND SEVERAL GATE OPENINGS

all power-efficiency curve at an unknown scale. The tangent point of the envelope to the individual curves at the several blade angles A to E determines the proper gate opening at these blade settings. If, as the author suggests, the peak efficiency is estimated, the efficiency curve is then determined, as well as all necessary data for the proper operation of the unit. Thus, the advantages of a field test may be obtained on a unit where it is impossible to make an accurate water measurement. Such an index test may be made on any plant at a saving in cost over a more extensive one in which a water measurement is made.

E. B. STROWGER.¹⁴ The author shows that at Safe Harbor the exponent a in the equation representing the Winter-Kennedy deflection-discharge relation was determined by experiment to be 0.5, this equation being $Q = C \times D^a$. With a equal to 0.5 it is apparent that, in the case of the Safe Harbor units, the force due to the piezometer differential varies with the square of the flow. Since centrifugal force varies as the square of the speed, the author was able to utilize an integrator carrying a flyball system, so arranged that the centrifugal force due to rotation of the integrator is opposed to the force of a tilting mercury manometer, resulting in a direct linear relationship between flow and integrator speed. If the value of the exponent had been found to be other than 0.5, the relationship between flow and integrator speed would not be linear and the integration of the flow would have been more complicated. Possibly in this case a slight correction in the flyball system could be made to produce the desired direct relationship.

Table 4 of this discussion is presented to show a number of Winter-Kennedy tap calibrations which have been made by the Gibson method of testing on 50 units in 12 different hydroelectric power plants. The exponent a for these taps is shown to vary from a minimum of 0.476 to a maximum of 0.538 and the arith-

¹⁴ Hydraulic Engineer, Buffalo, Niagara and Eastern Power Corporation, Buffalo, N. Y. Mem. A.S.M.E.

¹¹ "Changing Requirements in Hydraulic Turbine Speed Regulation," by F. Nagler, Trans. A.S.M.E., vol. 52, 1930, HYD-52-2.

¹² Principal Mechanical Engineer, Tennessee Valley Authority, Knoxville, Tenn. Mem. A.S.M.E.

¹³ Assistant Chief Engineer, S. Morgan Smith & Company, York, Pa. Mem. A.S.M.E.

TABLE 4 RATINGS OF CERTAIN WINTER-KENNEDY TAPS BY THE GIBSON METHOD

$$(Q = C \times D^a)$$

Plant	Unit	Date of Test	No. of Test Runs	Taps Used	C	a
1	1	6/18/31	26	-	865.9	0.529
2	4	9/10/31	45	-	1923.3	.505
3	1	9/25/30	45	-	949.0	.503
4	1	4/21/32	29	-	34.1	.500
4	2	4/22/32	29	-	34.1	.500
4	3	4/23/32	29	-	34.1	.500
5	2	3/15/33	30	(R4-R1)	2759.9	.508
5	2	3/15/33	30	(R4-R2)	3313.1	.508
5	9	3/11/33	30	(R4-R1)	2644.3	.518
5	9	3/11/33	30	(R4-R2)	2989.8	.518
5	A	3/13/33	31	(R4-R0)	557.9	.521
5	A	3/13/33	31	(R4-R1)	599.5	.521
5	11	8/16/34	32	-	2734.0	.513
6	1	8/15/33	26	(R4-R2)	646.9	.535
6	1	8/15/33	26	(R4-R3)	739.1	.535
6	2	8/16/33	25	(R4-R2)	690.1	.535
6	2	8/16/33	25	(R4-R3)	794.9	.535
6	3	8/17/33	25	(R4-R2)	668.1	.538
6	3	8/17/33	25	(R4-R3)	740.3	.538
6	4	8/18/33	25	(R4-R2)	647.8	.538
6	4	8/18/33	25	(R4-R3)	753.1	.538
7	5	9/18/33	42	(Note 1)	2728.9	.496
7	2	10/22/35	47	(Note 2)	1212.9	.521
8	1	4/16/37	30	(R2-R1)	977.5	.508
8	1	4/16/37	30	(R3-R1)	835.7	.508
8	1	4/16/37	30	(R4-R1)	886.0	.508
8	2	4/19/37	31	(R2-R1)	1012.2	.503
8	2	4/19/37	31	(R3-R1)	893.0	.503
8	2	4/19/37	31	(R4-R1)	710.9	.503
8	3	4/21/37	30	(R2-R1)	1029.6	.495
8	3	4/21/37	30	(R3-R1)	870.9	.495
8	3	4/21/37	30	(R4-R1)	732.8	.495
8	4	4/23/37	22	(R2-R1)	1035.5	.505
8	4	4/23/37	22	(R3-R1)	894.5	.505
8	4	4/23/37	22	(R4-R1)	721.2	.505
9	3	10/12/37	35	(R5-R2)	817.1	.508
9	3	10/12/37	35	(R5-R4)	941.8	.515
9	8	10/15/37	35	-	354.1	.508
9	5	10/15/37	35	-	436.7	.476
9	5	10/11/38	21	(R6-R4)	873.1	.505
9	5	10/11/38	21	(R6-R3)	801.8	.508
9	5	10/11/38	21	(R6-R2)	724.6	.505
10	2	10/20/37	32	(R6-R1)	1310.5	.508
10	2	10/20/37	32	(R6-R3)	1653.7	.510
10	1	10/25/37	32	(R6-R1)	1354.6	.508
10	1	10/25/37	32	(R6-R3)	1704.8	.503
11	2	11/19/37	28	-	2653.7	.500
11	3	11/30/37	28	-	2614.2	.505
11	1	12/1/37	28	-	2628.4	.503
12	4	1/28/38	37	-	1709.7	.481

Note 1. Sum of three deflections. $(R4-R2) + (X2-R2) + (X1-R2)$

Note 2. Function of four deflections. $2(X1 + X2 + R4 + Y1 - 2R1 - 2R2)$

metic average of all values shown is 0.511. While the theoretical value of the exponent is probably 0.5, in many cases the character of the flow in the vicinity of the taps or the condition of the tap equipment may be such as to cause the value of the exponent to depart slightly from the theoretical value. Attention is particularly called to the values of a for unit 8 of plant No. 9 where the test on one set of taps showed a value of 0.508 and the same test on another set showed a value of 0.476.

I. A. WINTER.¹⁵ The writer has had occasion to check the performance of type 2 and type 3 flowmeters, and finds that both instruments are capable of a high degree of accuracy and reliability with, apparently, the advantages of integration slightly in favor of the type 3 meter and the advantages of indication and servicing slightly in favor of the type 2 meter. That part of the paper which the writer is best qualified to dis-

cuss is the performance of the prime mover or the differential pressure taps, located on opposite sides of the turbine scroll case, of which considerable data have been accumulated.¹⁶

The Winter-Kennedy piezometer system, shown in Fig. 1 of the paper, depends upon the effect of centrifugal force of the water as it flows about the vertical axis of the unit, and therefore registers as a function of the flow past the piezometer section only and is not affected by the coefficient of friction of the walls of the conduit, the angle of the turbine gates, or the head on the power plant. Whenever possible, pertinent data relating to the performance of the taps, with respect to these factors, have been obtained and it may be said that the results have been highly satisfactory.

An example of the comparison of performance of the prime mover and the type 2 meter is illustrated in Fig. 18 and Table 5

¹⁶ Ref. (2) of paper.

TABLE 5 PERFORMANCE OF DIFFERENTIAL-PRESSURE TAPS AND FLOWMETER UNDER VARYING HEAD

Servo-motor-piston stroke, in.	d In. mercury at 524-ft head	d Reduced to 453-ft head by Equation [12] ¹⁶	d From curve for 453-ft head	Q At 524-ft head determined by manometer	Q By flowmeter dial, type 2	Departure, per cent
0.00	0.00
2.17	0.55	0.48	0.46	330	335	+1.5
3.08	1.19	1.03	0.99	485	480	-1.7
3.97	2.15	1.86	1.85	640	636	-0.7
4.86	3.36	2.91	2.91	775	790	+1.8
5.77	5.08	4.40	4.40	935	940	+0.5
6.67	6.56	5.68	5.83	1070	1082	+1.2
7.33	7.85	6.79	6.98	1165	1175	+0.8
8.02	9.15	7.92	8.23	1255
8.87	10.68	9.24	9.60	1350
8.45	10.20	8.82	8.95
7.56	8.36	7.23	7.41
6.67	6.73	5.82	5.83	1080
5.98	5.40	4.67	4.67	970
5.31	4.26	3.69	3.69	865
4.42	2.75	2.38	2.38	710
3.53	1.74	1.51	1.35	580
2.26	0.88	0.76	0.50	425
0.00	0.00

No readings taken

NOTE: The original calibration of the flowmeter taps was made October 15, 1937, at a gross head on the plant of 453 ft. The meter register was calibrated to agree with these data. The check test by the index method was made on December 27, 1939, and the head on the plant at that time had increased to 524 ft.

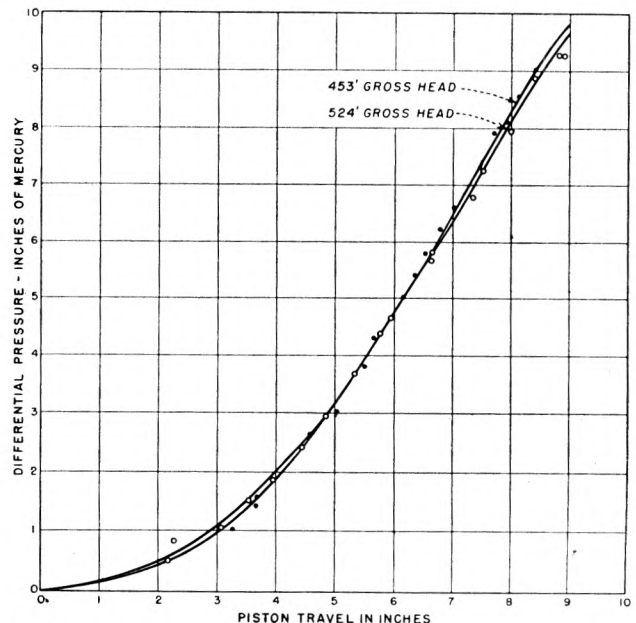


FIG. 18 RELATION OF TURBINE-SERVO-MOTOR-PISTON TRAVEL TO DIFFERENTIAL PRESSURE CORRECTED TO A COMMON HEAD OF 453 FT

¹⁵ Senior Engineer, United States Bureau of Reclamation, Denver, Colo.

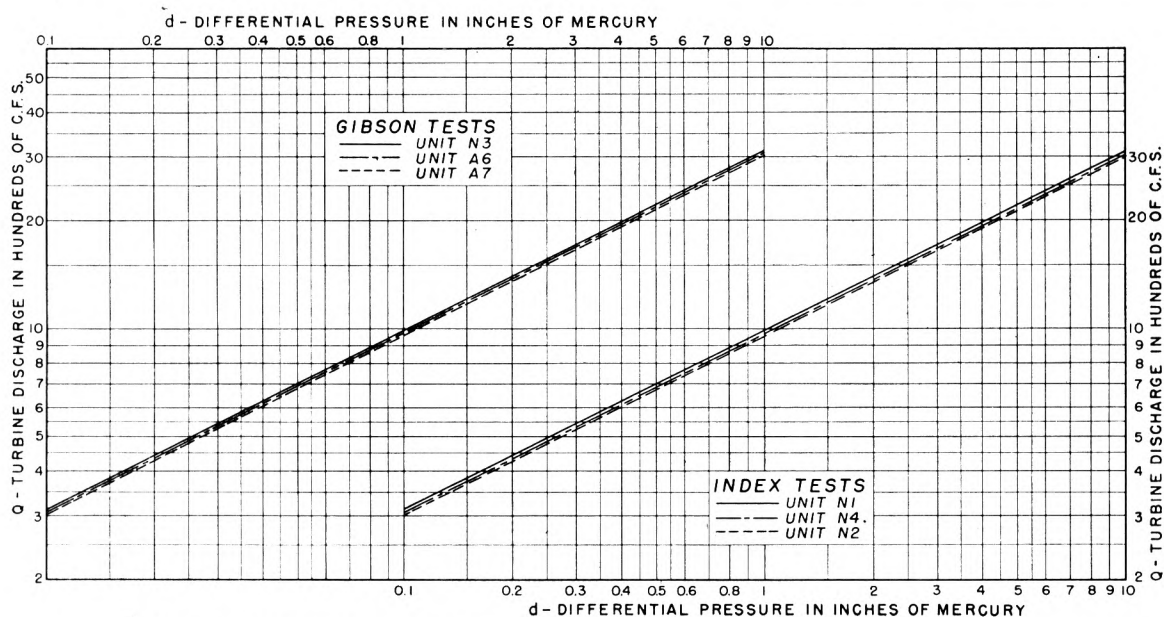


FIG. 19 COMPARISON OF FLOWMETER PERFORMANCE FOR IDENTICAL TURBINE-SCROLL CASES

TABLE 6 FLOWMETER CONSTANTS AND DEVIATIONS FOR FIG. 19

Unit	Constant c in equation $Q = cd^{0.5}$	Deviation from average $c = 972$ for 6 units, per cent
N1	988	+1.6
N2	953	-2.0
N3	986	+1.4
N4	968	-0.4
A6	976	+0.4
A7	962	-1.0

of this discussion, showing the relation of turbine-servomotor-piston travel to differential deflection in inches of mercury, as observed on differential-pressure taps, installed in unit A-8 at the Boulder power plant. The curve for gross head of 453 ft was determined by Mr. Gibson, using the time-pressure method of water measurement. The curve, designated as 535-ft gross head, represents comparable measurements when reduced to a common head of 453 ft in accordance with Equations [11] and [12],¹⁶ stating that the deflection readings may be reduced to a common head directly as the ratio of the common to test heads or H_c to H_t . This curve shows very good agreement between the original calibration made in 1937 and the check test made in 1939. The maximum deviation of these curves is about 2 per cent, which is to be expected, since there is a change in the coefficient of discharge of the turbine, due to the constant speed of the runner under varying heads and, also, there is a change in the coefficient of discharge of the turbines in the opposite direction, due to the roughness of the turbine orifices increasing in the two-year period between tests, thereby lowering the coefficient of discharge. The index test at 535-ft head included readings of the flowmeter dial, for comparison with the observed quantities determined by the manometer readings shown in Table 5. This table shows the maximum deviation between the flowmeter dial and the manometer reading to be plus $1\frac{1}{2}$ per cent and the average throughout the range of the curve is less than 1 per cent. This would appear to be a satisfactory performance of the meter after two years of continuous operation and without special adjustment for the tests.

Fig. 19 of this discussion is of special interest in comparison of the discharge-differential-pressure relation as obtained on units of the same design. This figure shows results of calibration made on six units with identical scroll cases at the Boulder power plant, each developing in excess of 115,000 hp, when operating at a

head of 475 ft or higher. Three sets of flowmeter taps were calibrated by the time-pressure method of water measurement (Gibson tests), and three sets were calibrated by the index method, using one unit tested by Mr. Gibson as a basis and assuming the other units to have the same coefficient of discharge.

It is not to be expected that the performance of the taps for similar units will be in better agreement than the data shown in Fig. 19, due to the lack of exact cross-sectional area of the casings at the metering section, and the lack of similarity of the runner and turbine gates which affects the coefficient of discharge at different points around the turbine speed ring. It is also likely that the coefficient of discharge may vary as much as 5 per cent for similar turbines, due to differences in orifice areas in the runner, inherent with the difficulties of producing large steel castings. The change in discharge of the unit due to a change in the controlling-turbine areas may be accounted for by making precise calibrations of the runner, in order to apply the proper correction factor because of the lack of similarity when extrapolating test data from similar units.

A study of accumulated data for various tests of turbine-flowmeter installations indicates that exponent a of 0.5 gives more consistent results than exponents determined by other flow means. This is in agreement with the author's conclusion, based upon the study of results obtained by means of precise current-meter measurements. With these data as precedent, it is recommended that exponent a , used in the flowmeter calibrations, be accepted as 0.5 and the constant for each individual run be determined on this basis with the general equation for the entire range of flow determined by means of weighted averages so that the greater degree of accuracy obtained for the higher water measurements may be given full weight in the final equation. The data given in Fig. 19 are determined upon this basis.

AUTHOR'S CLOSURE

In making more pertinent data available, the various contributions to the discussion constitute a valuable addition to the paper. Mr. Borden's comments confirm the conclusions regarding the type 2 meter, as the amplitudes of the sinusoidal-error curves for the two meters of this type investigated are of the same magnitude as for the meter mentioned by Mr. Borden.

While the amplitude for one of the type 2 meters was found to be $(0.63 + 0.38)$ or 1.01 per cent, that of the other was $(1.39 + 0.82)$ or 2.21 per cent. Based on the 90-min test period and readings at 5-sec intervals, Mr. Borden's meter, of the same type picked at random, apparently has an error amplitude of at least $(0.8 + 0.9)$ or 1.7 per cent.

Regarding Mr. Bristol's contribution, it may be pointed out that the type 1 meter may also lend itself to efficiency-indicating-and-recording apparatus, as the electrical impulses obtained from the mercoird switches limiting the swing of the drum, Fig. 20 of this closure, could be used for the positioning of slide-wire elements. At the same time, it should not be lost sight of that

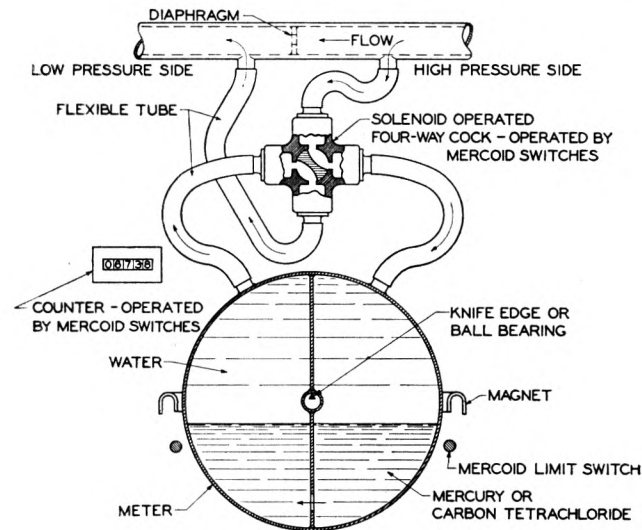


FIG. 20 SCHEMATIC VIEW OF TYPE 1 METER

load-limit indications, based upon hydraulic input or efficiency-indicating-and-recording apparatus, are not the only two alternatives to improve operating efficiencies. Alarm devices, similar in principle, may be based upon power output rather than upon hydraulic input and, if need be, with manual or automatic adjustment for head. Still other alarm devices may take advantage of gate opening using the stroke of the servomotor piston of the gate mechanism as a primary element.

Mr. Nagler touches upon a subject of wide interest. For many years there have been two schools of thought. One of these advocates that only vital machinery together with the very minimum of auxiliary equipment be installed with the intent of simplifying operation and keeping the investment as small as possible without impeding service. Those adhering to this point of view usually claim that many of the more modern power stations are over-equipped with auxiliary instruments and apparatus of all sorts, complicating operation unnecessarily. The second school and no doubt the more progressive one believes that the omission of certain apparatus is false economy and that auxiliary instruments are justified, provided they either pay their own way through increased generation or by making data available which may ultimately lead to improved service through furnishing the basis for performance investigations and thus contribute to an advancement in the art. It is thought, however, that no single

policy may be advocated as, depending upon local conditions, either course may be well justified.

Mr. Roberts very aptly points out that the metered turbine discharge will be invariably larger than the discharge determined indirectly, based upon output. Since this difference is inherent with the convex shape of unit-efficiency curves, it seems rather puzzling why any operator should object to metering.

Regarding Mr. Scoville's discussion, it is believed that the best speed-ring design is very likely also the most favorable to the installation of Peck piezometers. The most advantageous shape of the stay vanes necessarily is such that it produces no vortices which would adversely affect the turbine efficiency. It is under these ideal conditions, free of vortices, that piezometers, in this case those of the Peck type, are most reliable.

Index testing was not particularly stressed as it had been discussed in detail in a previous paper⁴ to which reference was made in this paper. However, a calibration of the piezometers by means of a quantitative water-gaging method, suitable to local conditions, is thought to be essential for a complete field test. Exclusive reliance on a step-up formula from laboratory results is as yet not feasible if a high degree of accuracy is desired for input data obtained by means of piezometers. With duplicate units of identical manufacture, the piezometer calibrations of at least one unit should be obtained by means of absolute water measurements, with the taps of the other units to be calibrated indirectly by assuming the efficiencies of both to be alike.

The question raised by Mr. Strowger for a possible adjustment on the type 3 meter for exponents differing by a small amount from 0.5 has been answered by Mr. Bristol. An adjustment is possible on the flyball system.

The statement made by Mr. Winter, regarding the performance of type 2 and type 3 flowmeters is interesting, particularly so because our experience has been different in that the type 3 meter was found to be not only more accurate but, at the same time, needed considerably less servicing than the type 2 meter.

In view of the conclusions reached with respect to the magnitude of the exponent, the additional piezometer-calibration data made available by Messrs. Roberts, Strowger, and Winter are of utmost importance. It is believed, however, that equal weight should not be given to each individual piezometer calibration cited. While some of the calibrations were obtained by means of graphic procedures, others were arrived at by means of analytical computations, using the method of least squares. Only the latter method is thought to be sufficiently accurate to warrant serious consideration, if a high degree of accuracy is desired. At the same time, the calibration method employed must be given some weight, together with the number of test points, magnitude of test-point dispersion inherent in the various absolute water-measuring methods, as well as local conditions. In this connection, attention may be drawn to the remarkable consistency of the results obtained with the water-gaging method employed at Safe Harbor, as it compares most favorably with that of any other method known at this time. Whether or not the exponent is actually 0.5 or a value very close to it may depend to a certain extent upon local conditions but there seems to be an agreement that for practical purposes an exponent of 0.5 may be entirely adequate in most cases from a metering point of view.

Speed Regulation of Kaplan Turbines

By J. D. SCOVILLE,¹ YORK, PA.

In this paper hydraulic-turbine speed-regulation data are presented and compared with calculated performance. Field tests made at the Bonneville and Guntersville power plants are cited. The effect of air admission during load rejection is discussed. Runaway-speed tests in the field are compared with model data.

THE problem of speed regulation has been the subject of numerous publications, although but meager field-test data have been presented for comparison with theoretical performance. It is the purpose of this paper to present the results of tests made on the adjustable-blade or Kaplan turbines at the Bonneville and the Guntersville hydroelectric plants, two recent power projects developed, respectively, by the United States War Department and by the Tennessee Valley Authority.

Kaplan turbines require special consideration in the matter of speed-regulation studies for three reasons:

- 1 The blades of the runner change pitch simultaneously with the movement of the gates for change of load.
- 2 The runner must frequently be placed below tail water to prevent cavitation.
- 3 The runaway speed is higher than that of other types of turbines.

Obviously, if the machine remains connected to a large system after a load change, the frequency variation cannot be calculated, since the flywheel effect of all the connected rotating machinery is one of the determining factors and is unknown. Consequently, when load is rejected from or imposed on a hydroelectric unit, its regulation must be calculated either on the basis of being connected with rotating machinery of limited extent and of known characteristics or on the basis of an isolated generator, operating on a load with no contributing WR^2 effect. The tests to be described were made in such a way that only the WR^2 of the rotating elements of the unit itself affected the regulation. When the load of a hydrogenerator is changed, the speed change depends upon the mechanical characteristics and condition of the governor, the flywheel effect of the turbine and generator, the hydraulic conditions at the plant, and certain hydraulic characteristics of the turbine itself.

CALCULATING SPEED DROP

The speed drop for a sudden increment of load on a unit is

$$S_d = \frac{81,000,000 \times HP \times T}{WR^2 \times N^2}$$

where S_d = speed drop, per cent

HP = load change

T = time of gate movement, sec

WR^2 = flywheel effect of rotating elements, lb-ft²

N = normal speed of unit

The product of this formula is an approximation, since in it the assumption is made that the input to the generator, during the

transition period, increases in a straight line. This is not quite true, but the error is not serious.

$$S_r = \frac{S_d}{1 + S_d/(N_r - 1)}$$

where S_r = speed rise for same load as given, rejected, and with the same time of gate movement

N_r = runaway speed expressed in relation to normal speed

If the turbine is operating in an open flume, the regulation calculated from the foregoing formulas will apply. If there is a closed channel or penstock leading to the wheel, pressure changes will occur during the gate movement and the speed rise or drop will be increased. The average pressure change during the load change should be used.

$$S'_d = \frac{S_d}{(1 - \Delta H)^{3/2}}$$

and

$$S'_r = S_r(1 + \Delta H)^{3/2}$$

where S'_d = speed drop, corrected for pressure drop

S'_r = speed rise, corrected for pressure rise

ΔH = average pressure drop, or rise as the case may be, expressed as a decimal

The maximum pressure change can be obtained (1)² by the use of the Allievi charts or other methods. This maximum pressure rise should be used in the design of penstock and casing, but for the purpose of correcting speed regulation, the average pressure rise should be used. Hence, T is the total time of gate movement when applied to the Allievi charts. The actual change in discharge should, of course, be used to compute the change in velocity. The momentum formula is also applicable in obtaining the average pressure rise, where

$$\Delta H = \frac{LV}{TGH_0}$$

For partial load changes the time of gate movement will not be in proportion. There is a "dead time" which includes the time to transmit the speed change from the flyballs to the pilot valve, to the relay valve, to the gate servomotor and, in the case of the Kaplan, to the blade servomotor, also to accelerate the oil in the piping and the mass of the gate mechanism. This should not be confused with the sensitivity of the governor. Table 1 shows approximately how the governor time varies with load.

TABLE 1 VARIATION OF GOVERNOR TIME WITH LOAD

Gate per cent	Governor time, sec						
100	1.5	2.0	3.0	4.0	6.0	8.0	
75	1.35	1.7	2.5	3.2	4.7	6.2	
50	1.2	1.5	1.9	2.4	3.4	4.4	
25	1.05	1.2	1.4	1.7	2.2	2.7	
10	0.95	1.0	1.1	1.2	1.4	1.6	

It is recognized that the formulas given are approximate. If a more exact method of computation is desired, the step-by-step method of computation may be used, in which the energy transfer between the water column and the runner is considered during short intervals of time during the load change. In using this method account must be taken of the relationship between the blade and gate movements and also of the lag of the blades, caused by the necessity of accelerating the blade mechanism and the oil

² Numbers in parentheses refer to the Bibliography at the end of the paper.

¹ Assistant Chief Engineer, S. Morgan Smith Company. Mem. A.S.M.E.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

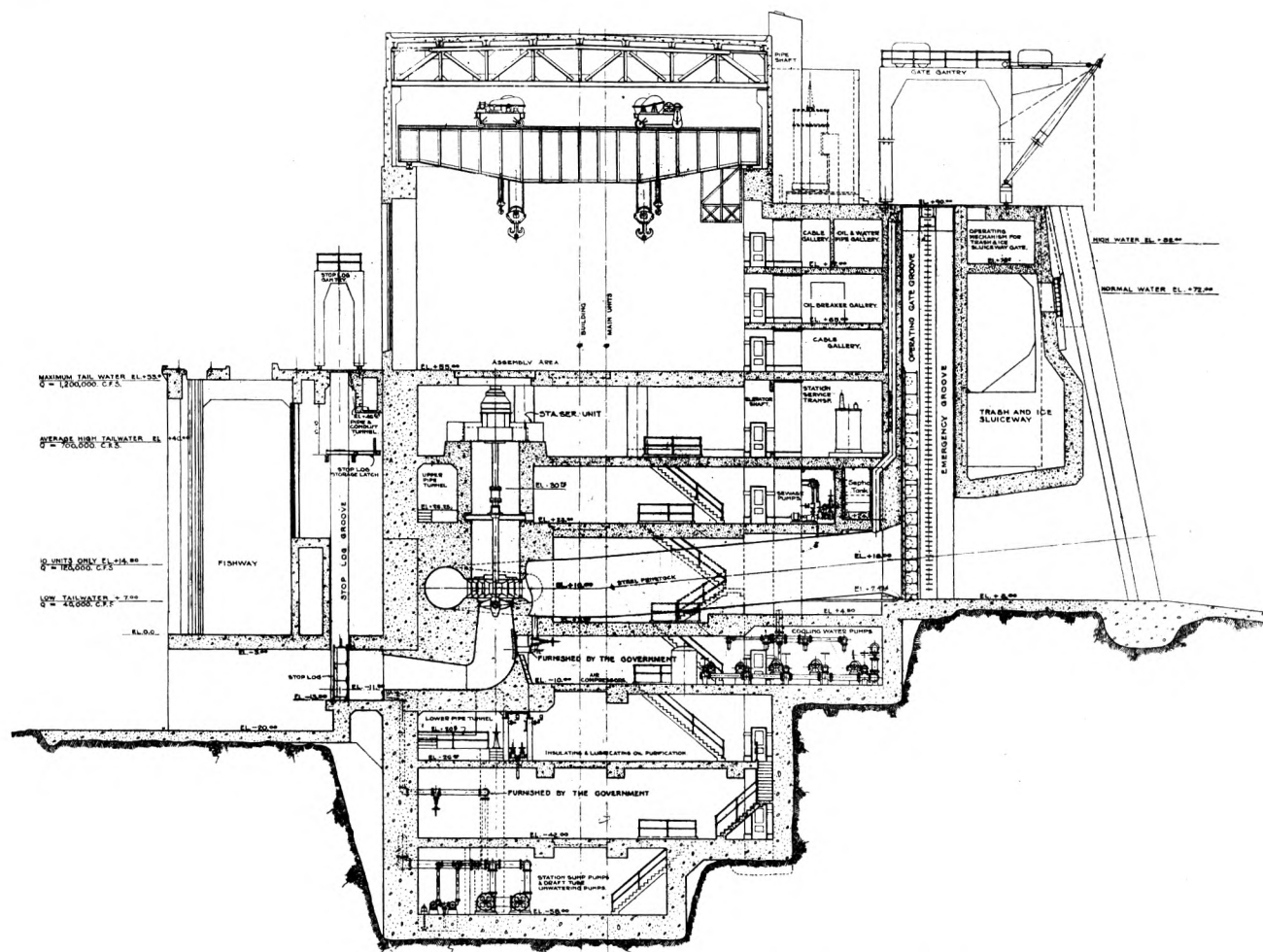


FIG. 1 CROSS SECTION THROUGH STATION SERVICE UNIT, BONNEVILLE PROJECT

in the piping. For a detailed discussion of this method, reference is made to a paper (2) by Strowger and Kerr.

APPLICATION OF METHOD TO BONNEVILLE TURBINES

The formulas and data have been applied to the Bonneville turbines and compared with field-test data obtained during the initial period of operation. These tests were made by the turbine manufacturer in cooperation with the United States Army Engineers, by whom this project was built. At that time, 1938, there were two Kaplan turbines of 66,000-hp capacity, driving 48,000-kva generators and one 5000-hp Kaplan wheel connected to a 4000-kw generator. A cross section through the station-service unit is shown in Fig. 1. The main turbines are the highest powered Kaplan wheels in the world and operate under a maximum head of 69 ft. For this reason, a substantial submergence was necessary to prevent cavitation. Fig. 2 is a cross section through the powerhouse, which shows the intake, turbine, and draft tube.

WATER RHEOSTAT USED TO LOAD GENERATOR

During the early operating period, the transmission lines were not completed so that, in order to load the generators, it was necessary to build a water rheostat capable of absorbing about 50,000 kw (3). With this apparatus, both load-on and load-off tests could be made without the introduction of any flywheel effect other than that of the machine itself.

PROCEDURE FOR TESTING

The station-service unit was tested first. The blades of this runner can only be adjusted with the machine shut down. Therefore, tests of speed rise and drop were made at several blade positions. The blades were adjusted to a given position, the generator loaded to a predetermined output, and then the circuit breaker was tripped. The speed rise was recorded by a Horne tachograph which was connected electrically to the generator through a transformer and which, by means of a flyball-operated pen, recorded the variation of speed during the transition period. A recording Bristol pressure gage was attached to the penstock adjacent to the turbine scroll to obtain the pressure change during the gate movement.

After a load-rejection test was made and the data obtained, the machine was again brought up to normal speed and voltage, and the circuit breaker closed. Since the rheostat was in the same position, instantaneous load-on regulation data were thus obtained from the Horne tachograph and the Bristol pressure gage. Altogether 34 load rejections and additions were made, at five blade settings and several gate openings each. Fig. 3 summarizes the data. It will be noted that while the computed speed rise checks fairly well with the test results, the speed drop from test is substantially less than the computed value. In this connection, it should be noted that, when some load less than the maximum turbine output is imposed, the gates by overtraveling make available more power input to the generator to prevent

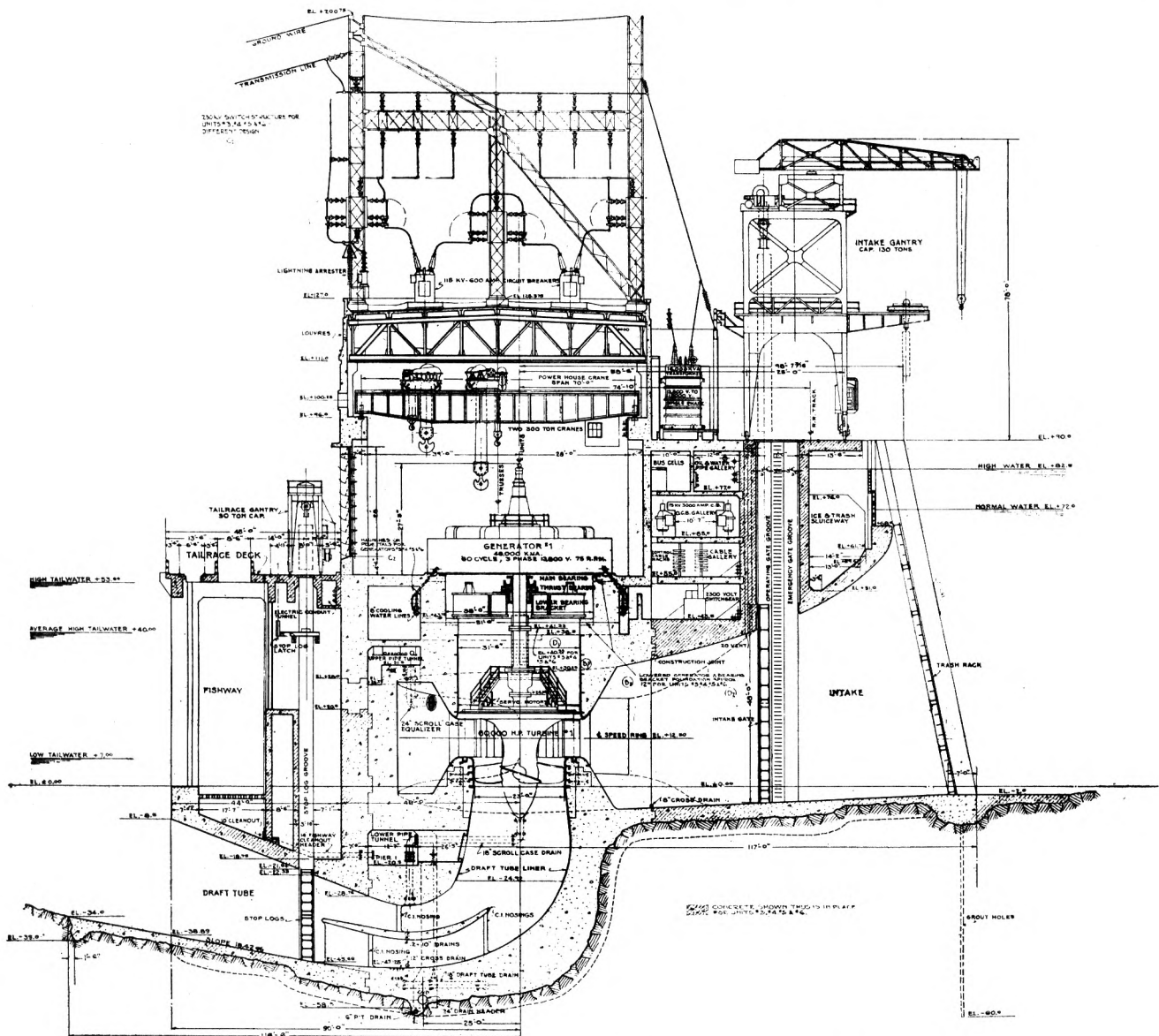


FIG. 2 CROSS SECTION THROUGH POWERHOUSE; BONNEVILLE PROJECT

speed drop; hence, the calculated value is on the safe side for all loads except for the maximum at each blade setting. Of course, it must also be realized that the mechanical condition of the governor has considerable bearing on the actual speed change. It should again be pointed out that, since the service-unit runner had manually adjustable blades, the tests cited were, in effect, on a fixed-blade runner at several angles.

TESTS ON MAIN UNITS

The main units are completely automatic Kaplans, that is, the blades move with the gates, taking predetermined positions for each gate opening. Tests were made on both of these units in the same manner as on the service unit, using the Horne tachograph and the Bristol recording pressure gage to obtain the necessary data. The water rheostat provided the load and was quite satisfactory except that, due to its location in the forebay, surges set up by the larger sudden load-on conditions caused severe power swings. More than 32,000 hp could not be taken on for this reason. Up to 60,000 hp, load rejections were made with no

trouble, since the surges occurred after the rheostat was separated from the line.

Fig. 4 shows the test results compared with the computed speed change and indicates fairly close agreement. The load-on regulation is relatively not quite so good as for the service unit. This can be explained by the fact that the blades do not move quite as rapidly as the gates and by the dead time in starting them. They are in the flat position before load is imposed and must open before any substantial amount of power can be developed. The blades of the service unit were open and ready to assume load.

It was found that, for large load rejections, the governor did not again come to rest without several oscillations. Fig. 6 illustrates this on unit No. 1, showing the record of speed change versus time taken by the Horne tachograph. As a matter of fact, the unit at first got completely out of governor control and the gates oscillated over their entire range and had to be stopped with the hand load limit. The reason for this is probably as follows: Since the runner is considerably submerged, after the gates

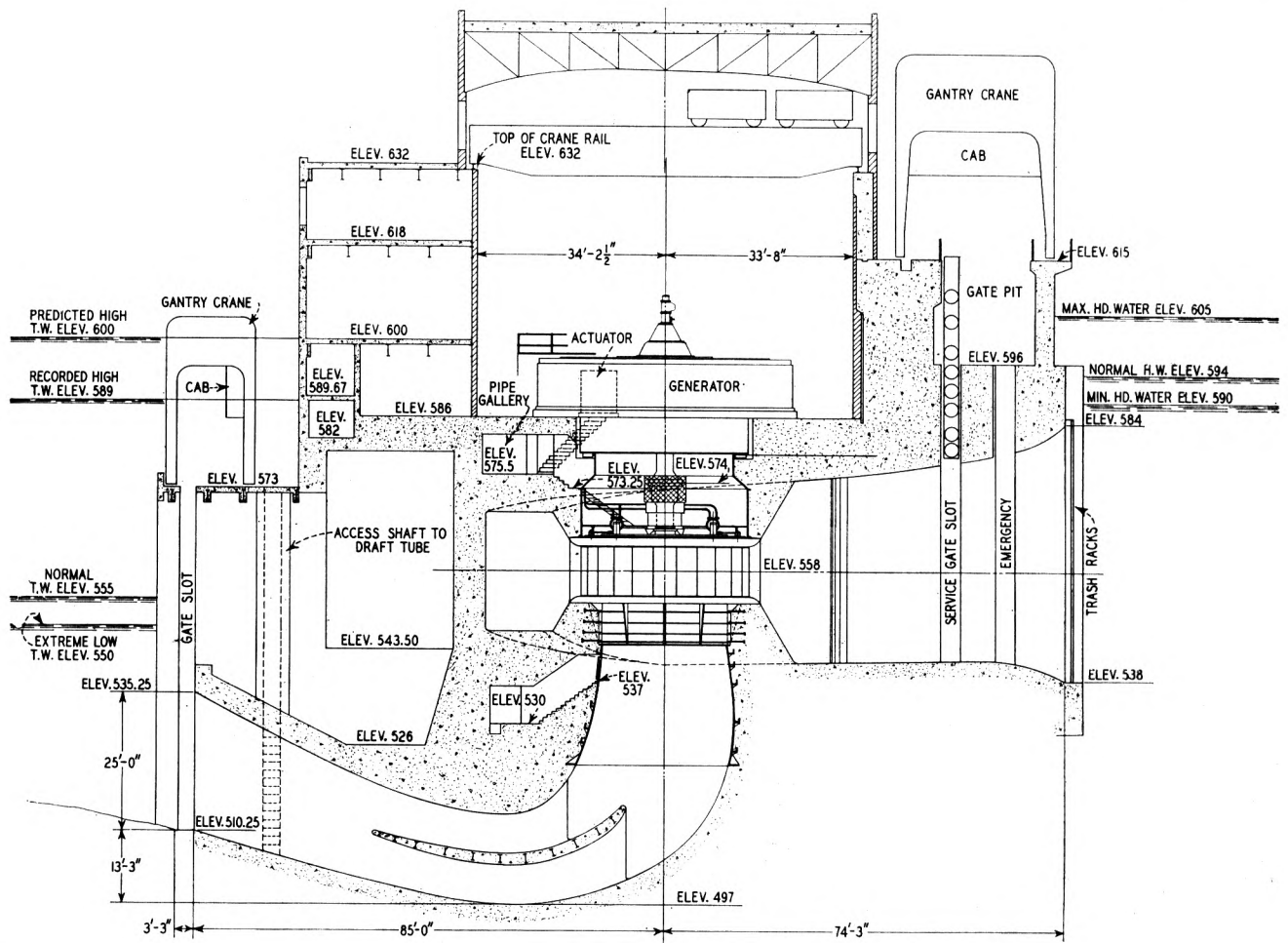


FIG. 5 SECTION THROUGH POWERHOUSE; GUNTERSVILLE PROJECT OF THE TENNESSEE VALLEY AUTHORITY

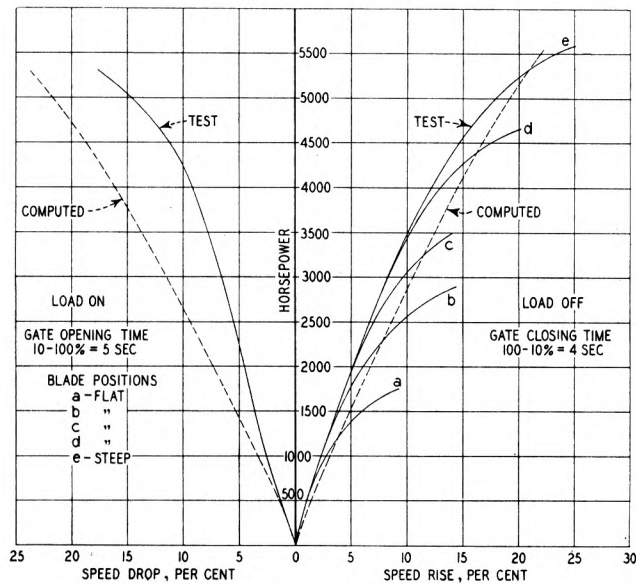


FIG. 3 SUMMARY OF LOAD-REJECTION AND ADDITION DATA

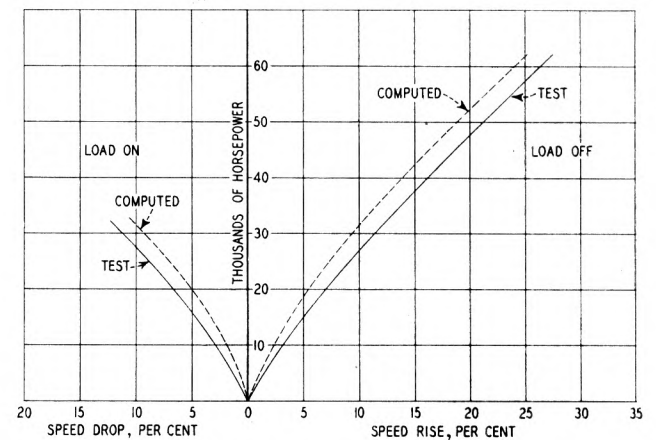


FIG. 4 LOAD-TEST RESULTS COMPARED WITH COMPUTED SPEED CHANGE

are closed, the runner operates as an axial-flow pump against a shutoff head equal to the barometric pressure plus the submergence. It attempts to pump the water between the runner and gates downward, and absorbs a large amount of power in doing so. This causes a rapid deceleration in speed, which requires a large gate movement and initiates hunting of the governor. This condition was much improved by the installation of a low-gate-limit stop which prevented the gates closing below the speed-no-load position when load was rejected, thus preventing the speed from reaching an excessively low value.

A slow closure at the end of the servomotor stroke aids the situation, if it is so designed as to permit the rapid opening of the gates the instant the governor so requires.

The introduction of air into the space between the runner and the gates destroys the vacuum there and reduces the braking effect on the runner so that the deceleration after the gates are closed is not so rapid.

TESTS AT GUNTERSVILLE PLANT

The foregoing was illustrated during tests on the turbines in the Gunterville plant of the Tennessee Valley Authority. These are Kaplan wheels of 42,000-hp capacity under 42-ft head at

69.2 rpm, driving 30,000-kva generators. Fig. 5 shows a section through one of the units in this plant.

As there was no water rheostat available, only load-rejection tests were made. The speed rise during load rejection was measured by the same Horne tachograph as was used at Bonneville. A Bristol recording pressure-vacuum gage was attached to the turbine top plate at A, Fig. 7. A spring-loaded air valve is connected at about the same elevation and so arranged that, when the vacuum under the top plate reaches a predetermined value, the valve opens and admits atmospheric air.

Fig. 8 shows the variation in pressure under the top plate at A on unit No. 3 during load rejections of from 7000 to 27,000 kw. It is shown here that, at the higher load rejections, a vacuum of 26 in. of mercury was reached during closure of the gates but that, by the admission of air, it quickly dropped to about 8 in.

A second load rejection of 27,000 kw was made on this unit, with the air valve closed, the effect being shown in Fig. 9. The vacuum reached 30 in. of mercury and then slowly dropped to a fluctuating value of 16 to 20 in., after an intermediate sharp drop to 8 in. It is this rapid change in vacuum which sometimes causes the rotating element to jump from the thrust bearing. The admission of air reduces this tendency considerably.

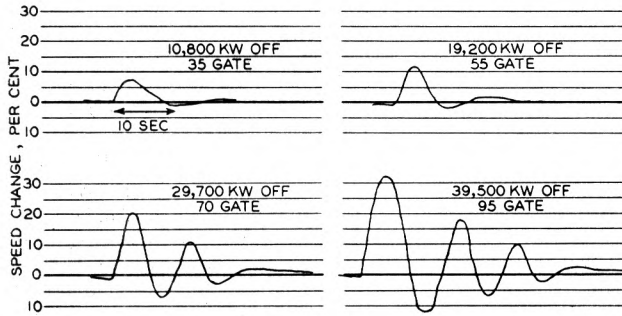


FIG. 6 RECORD OF SPEED CHANGE VERSUS TIME; UNIT No. 1

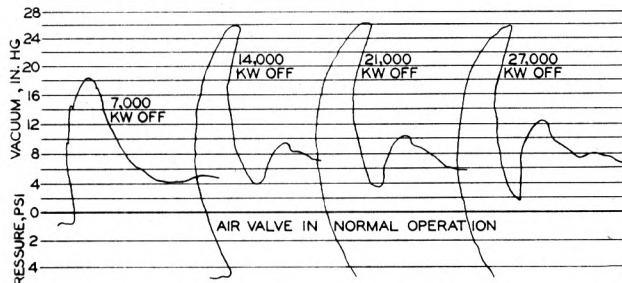


FIG. 8 VARIATION IN PRESSURE UNDER TOP PLATE AT A; UNIT No. 3, DURING LOAD REJECTIONS OF 7000 to 27,000 Kw

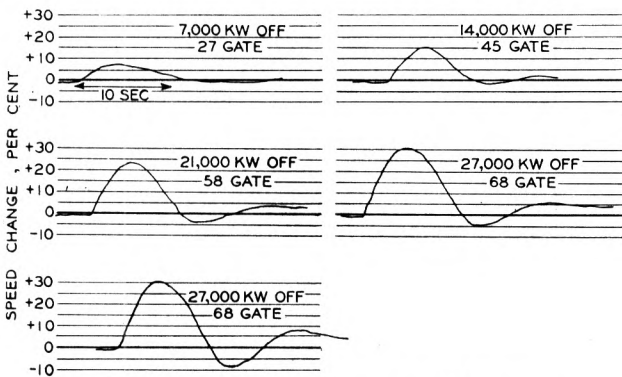


FIG. 10 SPEED-TIME CHARTS FROM TACHOGRAPH

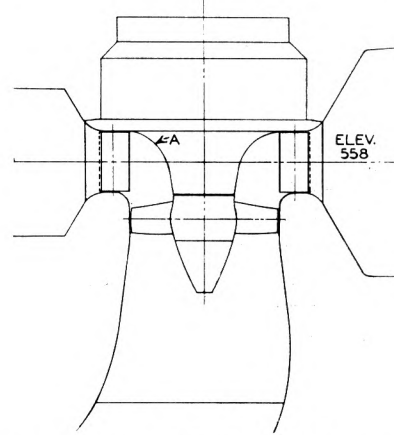


FIG. 7 RECORDING PRESSURE-VACUUM GAGE ATTACHED TO TURBINE TOP PLATE AT A

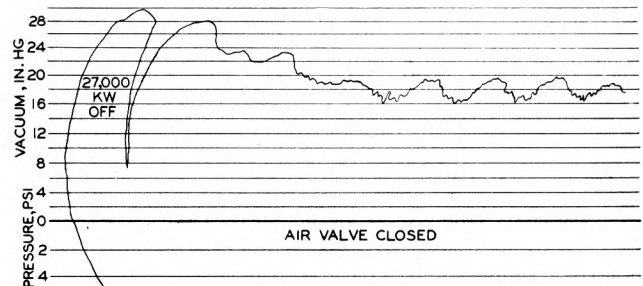


FIG. 9 EFFECT OF LOAD REJECTION OF 27,000 Kw ON UNIT WITH AIR VALVE CLOSED

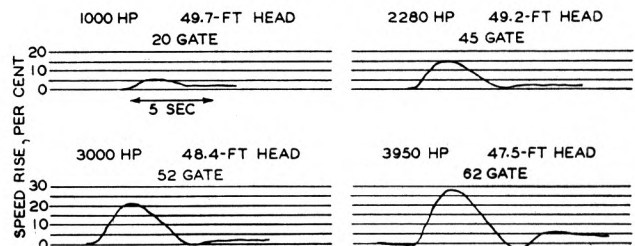


FIG. 11 EXAMPLE OF NORMAL SPEED REGULATION

The speed-time charts from the tachograph are shown in Fig. 10, the first four diagrams being for load rejections with the air valve in normal operation and the fifth curve for load rejection with the air valve closed. It will be noticed that the oscillation of speed below and above normal, after closure, is substantially less with the admission of air.

Fig. 11 is given as an example of normal speed regulation. This is a small unit not comparable in size with either Bonneville or Guntersville but shows the type of speed-time curves which are desirable. These are about the character of curves which would be obtained on a Francis turbine during load rejections.

In the event of governor failure, when the gates are wide open and, if for some reason, they remain in that position after the load has been tripped off, runaway speeds can be reached. Maximum speed occurs if the gates are wide open and the blades about half open. This is an abnormal condition, as the blades should be in the steep position at full gate. Under the latter condition, the overspeed is much reduced. The maximum speed which can be reached occurs at some intermediate gate-and-blade position and is about 85 per cent of the value of full-gate—half-blade position.

Overspeed tests were made on both turbines at Bonneville and the results checked within, 1 per cent and 5 per cent, the predicted values from model tests. It was found from these tests that runaway speed is affected by plant σ , where

$$\sigma = \frac{\text{Barometer} - (\pm H_s)}{H}$$

in which

H_s = suction head on runner; minus if runner is below tail water, and plus if above

H = head operating on turbine

During the runaway-speed tests in the field, it was noticed that there was a large opening tendency of the blades. This is a desirable feature since, at large blade angles, the overspeed is still further reduced.

CONCLUSIONS

The problem of speed regulation of large Kaplan turbines involves careful consideration of a number of factors which are not of particular concern in the ordinary Francis turbine. It is hoped that this paper has illustrated some of these and that more information will become available, as it is of interest not only to turbine and governor manufacturers but also to plant operators.

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Discussion

J. M. MOUSSON.³ This paper deals almost exclusively with load rejection. Although this phase of speed control is by no means the most important, it is perhaps the most spectacular. A great deal is claimed for air admission and yet, in the light of the data presented, there could perhaps be some doubt as to the effectiveness of air. Apparently, the author's case rests on a comparison between the two tests made with and without air admission at one 42,000-hp turbine of the Guntersville development under load rejection of 27,000 kw and shown in Fig. 10 of the paper. Based on these data alone, not much can be said in favor of air admission. While a 3 per cent gain in speed minimum may be observed with air, the speed maxima as well as the number of speed oscillations are identical in both tests.

Believing in the beneficial effect of air and to support the author's conclusion, some data obtained with an oscillograph on No. 3 unit at Safe Harbor in 1934 are presented. This turbine is rated at 42,500 hp under a head of 55 ft and operating at 109.1 rpm.

A comparison of the first two graphs in Fig. 12 of this discussion shows that, whereas, a load rejection of 18,000 kw without air injection produced 3 distinct cycles of speed oscillation, only 1½ cycles could be noted with air admission. This is indeed a striking example of the benefit of air injection, as it accomplished a 50 per cent reduction in speed oscillation and an almost equal reduction in time required to obtain speed equilibrium.

From a comparison of governor performance at Bonneville and Guntersville presented in Figs. 6 and 10 of the paper, respectively, it is evident that there must be either a fundamental difference in the governor design or in hydraulic conditions. Does the author attribute the improved performance at Guntersville to the gate-limit stop mentioned by him which prevents the gates from closing below speed-no-load position, together with provisions for a slow closure at the end of the servomotor stroke, so designed as to permit the rapid opening of the gates, if required by the governor?

While air injection did show very beneficial results at Safe Harbor, equally satisfactory governor-performance improvement was obtained with a cam mechanism installed on the governor of

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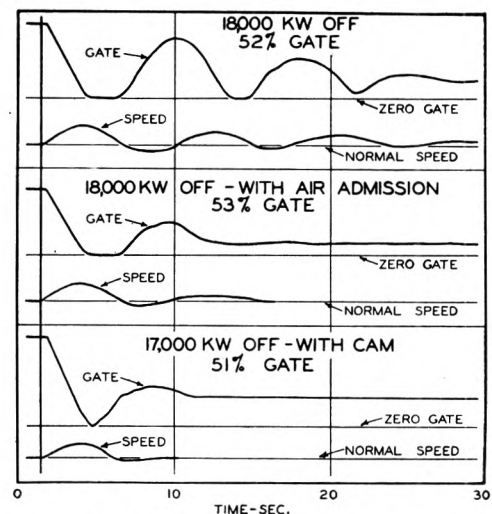


FIG. 12 SPEED AND GATE-OPENING CHARACTERISTICS UNDER LOAD REJECTION FROM OSCILLOGRAPH RECORDS

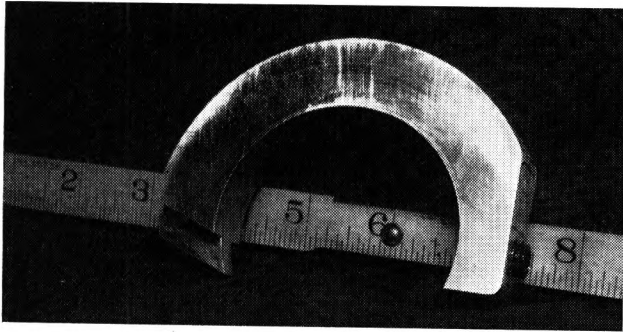


FIG. 13 SHAPE OF CAM TO PREVENT HUNTING

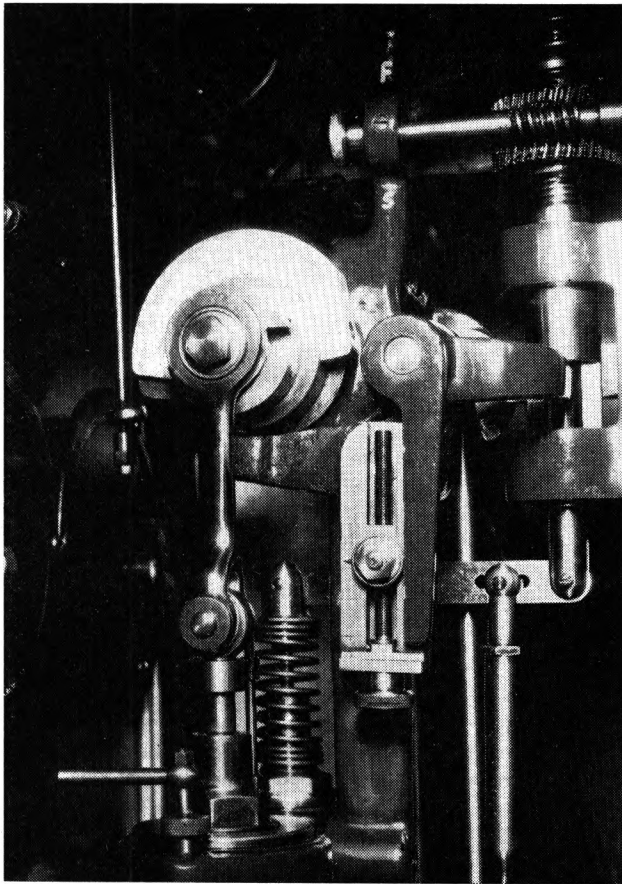


FIG. 14 CAM MOUNTED ON COMPENSATING CRANK OF SAFE HARBOR GOVERNOR

unit No. 3, introducing a large speed drop for the light-load range. The third graph in Fig. 12 shows the speed oscillation obtained with this device under load rejection of 17,000 kw. This cam device is mounted on the compensating crank, Figs. 13 and 14 of this discussion, in such a manner as to contact with the equalizing mechanism at a gate opening of 15 per cent on the closing stroke, causing an upward push on the speed rod and opening the pilot valve in the direction to open the gates. While the gates are still rapidly moving in the closing direction, forces are thus set up in the governor system opposing the closing motion, resulting in a dissipation of stored kinetic energy in the moving mass by the time the gates reach zero opening. The gates at once assume speed-no-load position with no further oscillation. In effect the cam increases the inherent speed drop

from approximately 2.5 per cent to approximately 10 per cent below 15 per cent gate.

After the tests on unit No. 3, all of the governors at Safe Harbor were equipped with a cam of this type. Subsequent experience in operation has fully justified their installation. It is believed that, while the method of air injection is satisfactory, its application as a continued feature to operation is not economical, due to the large reserve of compressed air required for emergency. The use of a cam mechanism or some other mechanical feature accomplishing the same results as described is preferable.

F. NAGLER.⁴ The type of comparisons made by the author are not encountered as frequently as they should be for the good of the hydroelectric-power field. These comparisons form an excellent answer to the criticism that the basic speed-change formula, given at the beginning of the paper, is only approximate. It is very definitely, however, a workable approximation, certainly as accurate as necessary for the purpose in question.

Three of the four variables in that formula, that is, $WR^2 \times N^2$, divided by HP , are constant for any particular unit and regulation depends principally upon them. It is, ordinarily, much simpler to speak of them as the regulating constant, since they represent the ability of a unit to regulate the speed. Expressed in the inverted form immediately preceding, they ordinarily vary between 5,000,000 and 10,000,000. It is of special interest that certain sizes and speed ranges of units must have additional flywheel effect added to the generator to raise this regulating constant to a feasible figure, whereas, other types, particularly in the high-speed and large-capacity ranges, inherently possess a regulating constant sometimes larger than is necessary. This is dictated by generator design.

Putting this regulating constant in the denominator of the author's basic speed-drop formula leaves the speed regulation dependent upon the governor time T . Modern governor guarantees mean little or nothing beyond the statement that, if the regulating constant is a certain amount, if the unit is disconnected from the load, and if a certain governor time is used, certain figures, which have practically nothing to do with the regulation of a system, can be computed.

Usually, the governor time, T , ranges somewhere between 2 sec and 10 sec. Most governors are purchased on the basis that they will close the gates in from 2 to 4 sec. Some of the largest operating companies, thereafter, normally adjust all of their governors so that they cannot close in a shorter time than 7 or 8 sec. Here we have a situation carried over from the days of isolated units, entirely comparable with insisting on a whip socket on the automobile dashboard. Bids may be compared on the basis of a few per cent difference in speed-change guarantees in the face of the realization, should thought be given to it, that these figures will probably be doubled in actual operation and probably with definite improvement to the system frequency.

The writer would suggest that, in the future, consideration be given to a simple statement of the regulating constant as an indication of the influence of the unit on system regulation and a statement of the guaranteed minimum governor time as an indication of the governor capacity. The absurdity of present-day governor guarantees will be somewhat reduced thereby.

Referring to the formula for average pressure rise immediately preceding Table 1, a somewhat closer approximation of the pressure rise and pressure drop, through the ranges usually experienced in practice, is obtained by adding 10 per cent to ΔH for pressure rises and subtracting 10 per cent from ΔH for pressure drops. This brings the approximate formula fairly closely in line with Allievi.

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It is significant that the author's charts of speed regulation show no results with load thrown on. This is logical because, in actual plant operation, except in connection with isolated units, as on a mining load, it is seldom that a unit ever has to meet sudden large increases in load. It is of interest that the author made tests with the load thrown on. The writer has used a water rheostat similarly, but was rather intrigued with the fact that, whereas, a load of, say 20,000 kw could be dropped instantly by pulling a switch, the same load was not picked up if the switch was immediately reclosed. There seemed to be a definite lag in the ability of the water rheostat to pick up the load it had just dropped. It would be of interest to have the author's comments on whether he observed any similar action in connection with load thrown on.

In conclusion, the writer would again emphasize the desirability of some change in the attitude of both purchasers and manufacturers in making governor guarantees. The formulas presented by the author have served their purpose for about 30 years, during which time probably 90 per cent of our turbine-horsepower capacity has become connected to the larger systems. Those systems operate the year round with 1 or 2 per cent maximum speed change. The units cannot receive large increases in load and, if they lose their load, they are disconnected from the system and have no influence on it. It may logically be concluded, therefore, that governor guarantees up to 25 or 30 per cent speed change are a clumsy attempt to effect a set of regulating conditions which may be much more simply and accurately expressed by regulating constant and governor time. Progressive purchasers are already looking at these basic figures, and greater progress would probably be made in the hydraulic-turbine field if more attention were directed toward them rather than to outmoded and academic speed-regulation tables.

J. F. ROBERTS.⁵ This paper covers quite fully the various factors which must be coordinated in order to obtain satisfactory regulation of Kaplan-type turbines. In the case of the Gunter'sville turbines of the Tennessee Valley Authority with which the writer was particularly interested, it was possible to vary both the rate of movement of the runner blades and of the wicket gates, and, thus, in the field, determine that combination which would result in the most satisfactory combination.

As finally adjusted, the wicket gates were set to open and close in from 8 to 10 sec with the runner blades opening in about 8 sec, but requiring about 40 sec to close. While faster operation of the wicket gates is possible, it was found that on such a large system the frequency of the system varies slowly even for a sudden loss of 30,000 kw. More rapid operation of the wicket gates imposes heavy loads and shocks on the wicket-gate mechanism without apparent improvement to the system-speed regulation. It also causes greater pressure variations, particularly in the draft-tube water column which, while possibly not dangerous to the structures, are unpleasant and, in this case, apparently unnecessary.

It is interesting to note that the author's tests demonstrated it to be almost impossible to obtain a flat blade angle of the runner at full wicket-gate opening and runaway speed. This means that on future projects it may be possible to make an appreciable saving in generator costs, since generators designed for full gate and flat-blade runaway must be designed for about 270 per cent of normal speed, as compared with about 200 per cent speed for full gate and steep pitch on the runner blades.

E. B. STROWGER.⁶ The author has presented test data on the

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⁶ Hydraulic Engineer, The Niagara Falls Power Company, Buffalo, N. Y. Mem. A.S.M.E.

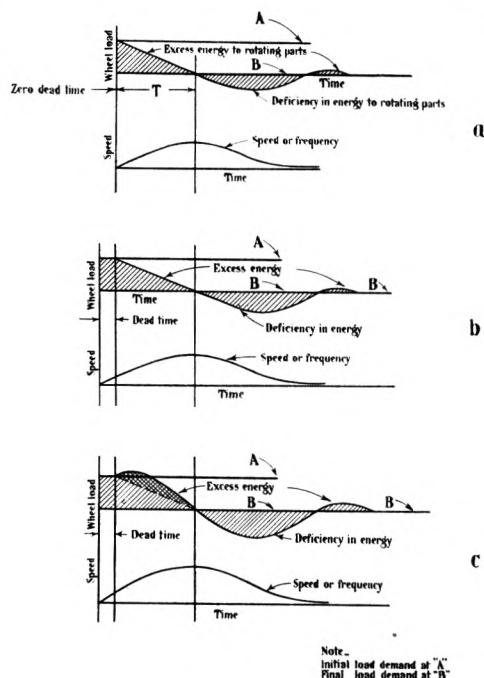


FIG. 15 EXCESS ENERGY CAUSED BY LOAD CHANGE

speed regulation of Kaplan turbines which are of value to hydraulic engineers engaged in water-power development, since but little if any such data have been published in the technical press to date. His approach has been through an approximate equation for reasons stated in the paper. In discussing the matter, he has referred to a paper (2) by S. L. Kerr and the writer, dealing with the step-by-step method of computing speed rise or speed drop for sudden load changes, in which a rational approach has been attempted and in which application of this method has been made to Francis turbines with good results. The writer therefore wishes to discuss the possibility of the application of this step-by-step method to Kaplan turbines.

Let us first consider a few simple but fundamental relationships. Referring to Fig. 15 of this discussion, assume an isolated unit operating on a load with little or no connected Wr^2 so that the only steadying influence on the speed would be the flywheel effect of the generator rotor. Suppose a load change takes place, as shown in Fig. 15 (a), the load dropping from A to B, and let us assume that the length of penstock and of draft tube is negligibly small. The turbine load then decreases approximately as a straight line with respect to time, as the governor closes the turbine gates to the new position of load demand. In this case, the excess rate of energy transfer from the penstock to the turbine above that demanded is gradually decreased as the governor moves the gates toward the new position and a rise in speed of the rotating parts takes place. This rise in speed actually makes the gates overtravel so that for a short time a deficiency of energy is transmitted to the rotating parts, followed by another small amount of excess energy, after which the travel of the gates has become damped to the extent that equality is again established between rate of energy supply and rate of energy demand.

In the case assumed, i.e., with the unit operating on an isolated load having no Wr^2 , the excess energy produced in the penstock must be balanced by the deficiency of energy, before the initial speed is re-established. It should be noted that, if no governor adjustment is made, the speed of the unit will not return exactly to its initial value, but the final value of speed will be slightly higher, depending upon the load dropped and the in-

herent speed-drop setting of the governor. This can also be seen in Figs. 6, 10, and 11 of the paper. The speed rises to a maximum value at the end of time T , as shown in Fig. 15 of this discussion. This maximum is not reduced by the subsequent overtravel of the gates either for loads off or for loads on. The overtravel, however, brings the speed back approximately to the normal value, after the maximum speed is reached faster than would be the case with no overtravel.

Fig. 15 (b) shows a similar load change with the introduction of the element of governor dead time, and indicates that dead time increases the excess energy which must be absorbed by the rotating parts and, therefore, results in a greater change in speed.

Now, passing to the usual installation where a physical length of penstock and draft tube must be considered, as illustrated in Fig. 15 (c), the rate of change in energy transfer from runner to rotating parts is not uniform and, consequently, the turbine load plotted with respect to time is not a straight line. In fact, the power produced by the turbine actually rises at the beginning of gate movement and then decreases as shown. That part of the excess energy shown double-hatched in Fig. 15 (c) is caused

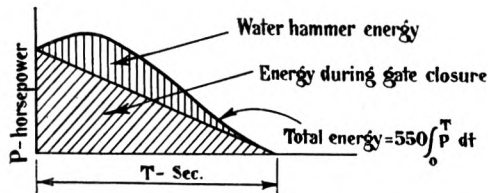


FIG. 16 SAME DIAGRAM AS FIG. 15 (c), EXCEPT GOVERNOR DEAD TIME IS CONSIDERED TO BE ZERO

by the water hammer, produced in the penstock and draft tube, due to the destruction of velocity, and may be a rather large percentage of the total excess which must be absorbed. The speed of the isolated unit changes, as shown on the diagram.

Now let us consider Fig. 16 of this discussion, which presents the same diagram as shown in Fig. 15 (c) except that the governor dead time is considered to be zero. The energy absorbed by a rotating mass in changing the speed from N_1 to N_2 may be expressed by

$$\frac{Wr^2}{5870} (N_2^2 - N_1^2)$$

The expression $550 \int_0^T P dt$ represents the total energy delivered from the water column to the runner during the gate closure and may be equated with the foregoing expression, representing the energy absorbed by the rotor. If the resulting equation is then solved, we obtain the following expressions for percentage of speed rise and percentage of speed drop.

$$\text{Speed rise, per cent} = \frac{N_2 - N_1}{N_1} \times 100$$

$$= 100 \left[\sqrt{\frac{3,229,000 \int_0^T P dt}{Wr^2 \cdot N_1^2} + 1} - 1 \right] \dots [1]$$

$$\text{Speed drop per cent}$$

$$= 100 \left[1 - \sqrt{1 - \frac{3,229,000 \int_0^T P dt}{Wr^2 \cdot N_1^2}} \right] \dots [2]$$

It should be noted that these expressions contain the same variables as the first equation given by the author but that the expression is radically different in form. The question now

arises as to how to evaluate the integral in these equations. For turbines with appreciable length of penstock, the evaluation of this integral depends upon the water hammer produced, the relation between runner efficiency and gate opening, the relation between runner efficiency and speed, and the rate of gate motion.

For open-flume settings and where the load varies in a straight line with respect to time, the integral may be closely approximated in the case of Francis runners by

$$\int_0^T P dt = \frac{1}{2} HP \cdot T \dots \dots \dots [3]$$

where HP = initial load in horsepower for speed rise and final load for speed drop

Consequently, for open-flume settings, these relations become for Francis wheels

$$\text{Speed rise} = 100 \left[\sqrt{\frac{1,614,000 HP \cdot T}{Wr^2 \cdot N_1^2} + 1} - 1 \right] \dots [4]$$

$$\text{Speed drop} = 100 \left[1 - \sqrt{1 - \frac{1,614,000 HP \cdot T}{Wr^2 \cdot N_1^2}} \right] \dots [5]$$

In the case of Francis runners, the step-by-step method of calculation lends itself to the evaluation of this quantity, because the power input to the runner can be calculated for each small interval of time during the gate motion. However, for Kaplan turbines, as pointed out by the author, there may be a lag of the motion of the blades behind that of the gates, which changes the steady-state gate-opening-efficiency relationship and, consequently, the quantity considered cannot be evaluated for this type of turbine without determining the effect of the lag and other factors which influence the efficiency of energy transfer from the water column to the runner during the gate movement. Possibly this effect can be determined experimentally for a number of Kaplan turbines and an average coefficient obtained. In the case of the Bonneville main unit, the writer attempted to do this for the full-load point on the curve of the author's Fig. 4, and obtained the relation $\int_0^T P dt = 0.39 \times HP \cdot T$. This showed, in the case of this particular Kaplan, that, even with some water-hammer energy present, less than 50 per cent of the energy represented by the product of HP and T was effective in speeding up the rotor during the load rejection. This means that the area under a straight-line curve of horsepower versus time represents too much energy transfer and shows the error, in one particular case, of the assumption made in deriving the approximate equation, i.e., a straight-line variation of these quantities during the transient.

To compare a typical Francis speed-rise problem with that of the Kaplan turbine, the writer made some computations for a Francis turbine having about 690 ft of penstock and obtained the figure of 0.61 as the coefficient of the quantity $(HP \cdot T)$ in Equation [4]. This is of the right order because for an open-flume setting of a Francis turbine, it would be reasonable to expect to obtain a coefficient of about 0.5.

As a check on the rationality of the equations presented here, the writer determined the coefficient of the product $(HP \cdot T)$ in the case of the speed-drop problem of the Bonneville service unit shown by the author's Fig. 3. A coefficient was determined by using the test data for a load of 5000 hp and then, since this unit had manually adjusted blades, the speed-drop points for 75 per cent gate and 50 per cent gate were computed using this coefficient, but applying estimated values of power for the two gate positions. A curve of speed drop was obtained, which checked the experimental curve at the part-load points within about 2 per cent and the shape of the curve was thus much closer

to the test curve of Fig. 3 of the paper than that of the computed curve of Fig. 3.

In conclusion it may be possible to apply the step-by-step method to Kaplan turbines by obtaining a graphic record of the movement of both the guide vanes and the runner blades during load changes and with the use of complete turbine characteristic curves, by evaluating the energy transfer from water column to rotor. As an alternative it may be possible to obtain average values of the coefficient as described for wheels of a given manufacturer or of a given design for load rejections at say 100 per cent gate, 75 per cent gate, and 50 per cent gate and these data would enable one to predetermine the speed-rise power curve similar to Fig. 4 of the paper for any projected installation, knowing of course the size of the unit, governor time, Wr^2 , etc. These suggestions are made in an attempt to apply a method of computation which takes into account the fundamental relations of energy transfer which take place during the load change between the penstock and the rotor.

AUTHOR'S CLOSURE

Mr. Mousson's discussion substantiates the author's contention as to the advantage of admission of air to a Kaplan turbine during load rejection. Fig. 14 shows an alternate method used at Safe Harbor to prevent the secondary speed and gate swings. It is believed, however, that this method is not as good as that of air admission. The author knows of at least one occasion where one of the Safe Harbor units jumped from the thrust bearing during load rejection. Air admission would eliminate that trouble. It should be pointed out that air is injected under pressure at Safe Harbor, whereas atmospheric air would serve the

same purpose. In spite of about 15 ft of submergence of the runner, atmospheric air was readily taken into the No. 3 unit at Bonneville and produced speed-change diagrams during load rejection exactly similar to those illustrated in Fig. 10 of the paper which were obtained on one of the Guntersville units. This is evidence that the secondary swings in speed which occurred on unit No. 1 were eliminated by the use of air on unit No. 3. It is believed that this answers Mr. Mousson's question.

Figs. 3 and 4 show speed drop as well as speed rise for load change on one of the Bonneville main units and on the service unit. Mr. Nagler states that there seems to be a definite lag in the ability of the water rheostat to pick up the load it had just dropped. The curves of Figs. 3 and 4 indicate somewhat less speed drop than was computed. There may be something in Mr. Nagler's statement. However, this was about as close to instantaneous load on regulation as could be obtained on the tests.

The author agrees with Mr. Roberts that it is highly desirable to be able to vary the rate of blade and gate travel independently.

Mr. Strowger wants to attack the problem of speed change by computing the water-hammer energy during gate closure and adding it to the normal energy put into the runner during this period. He is, of course, quite right in this as the energy which produces speed change is the sum of these two minus the increased generator windage and friction. However, as pointed out in the paper, there is a variable lag in the blade movement in addition to the dead time of the governor. Lack of certain and definite information makes it difficult to apply Mr. Strowger's method on Kaplan turbines.

Development of the Automatic Adjustable-Blade-Type Propeller Turbine

By R. V. TERRY,¹ NEWPORT NEWS, VA.

In this paper the design and testing of automatic adjustable-blade-type propeller turbine models are treated, followed by an analysis and discussion of the model-test results. Four installations are described, aggregating a rated output of about 44,000 hp in five units. Operating experience and results with field units are considered. Mention is also made of further development work now in progress. In conclusion a summary is given of the principal features of this new type hydraulic turbine.

INTRODUCTION

THE idea of adjusting the pitch of the blades of hydraulic-turbine runners to suit the load is relatively old. Such a runner was described² in 1867. For about 25 years development has been in progress on the Kaplan adjustable-blade propeller type turbine. That type has reached a high degree of perfection in employing an oil-pressure system, associated with the governor-pressure system, to operate the runner blades in synchronism with the gates. From the many published articles on adjustable-blade turbines, the advantages of varying the pitch of the blades are well known. The principal advantages are as follows:

- (a) Increase in capacity above normal.
- (b) Sustained efficiency over the larger part of the load range.
- (c) Reduction of the minimum head at which the turbine will develop power.
- (d) Greater stability of operation, particularly at low and intermediate loads.

In 1928, the author began studies for an adjustable-blade type of runner wherein the blades would move automatically with changes in flow through the runner, as well as with changes of speed and head. It was his rather radical idea³ so to pivot the blades with reference to their centers of pressure that they would tend to adjust themselves to the most efficient angle for each condition of gate opening and head for constant-speed operation.

MODEL TESTS

Surprisingly satisfactory results were secured with the first models built and tested in 1930, which encouraged further experimentation. Other groups of tests have been conducted at intervals for the last 10 years. Altogether, more than 200 tests have been made on about 30 different models, using about 15 sets of blades, some of which were altered as the test program progressed. The model tests were made on 16 $\frac{1}{2}$ -in.-diam runners under heads varying from 9 to 12 ft, in the Newport News Hydraulic Laboratory.

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² U. S. Patent No. 67,994, issued to O. W. Ludlow in 1867.

³ U. S. Patents Nos. 1,858,566 and 1,907,466.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

It was early found that the blades must be pivoted considerably ahead of their centers of area. In fact this was predicted from a study of the characteristics of airfoils and hydrofoils. In order that the blades might have a tendency to adjust themselves automatically and with minimum operative moments, it was also found desirable to pivot them slightly ahead of their centers of pressure. This resulted in a tendency for the blades to open at all times, at least in normal operation at or near the best efficiency conditions. As the test program progressed, certain definite trends and characteristics were found which influenced subsequent designs. As the development gradually unfolded, unproductive leads were encountered, as well as new and desirable characteristics disclosed.

It is assumed that those interested are somewhat familiar with turbine design and with some of the terms used in aeronautics dealing with airfoils. In Fig. 1 (upper left) is given typical velocity diagrams for an average blade section. Diagram w_1, c_1, w_1 is for the inlet edge of the blade, while diagram w_2, c_2, w_2 is for the discharge edge. Terms w_1 and w_2 represent the respective velocities relative to the blade and w_a is the vectorial average of w_1 and w_2 . The function $u_1 = u_2$ is the tangential velocity of the blade. Terms c_1 and c_2 are the absolute velocities at inlet and discharge, respectively.

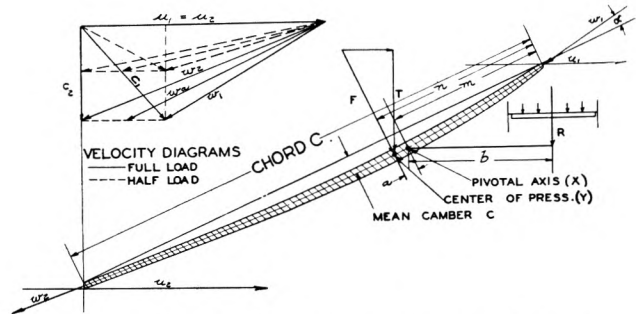


FIG. 1 TYPICAL BLADE SECTION AND VELOCITY DIAGRAMS

A typical blade section is also shown in Fig. 1, with chord C and mean camber c . The angle of attack α is the angle the inflowing water w_1 makes with the chord C . The center of pressure is shown at point y which is at distance n from the leading edge of the blade, measured along the chord. The pivotal axis is shown by point x which is at distance m from the leading edge of the blade. The equivalent total force on all the blades is represented by F , while T represents the axial component of F or the hydraulic thrust. The "hydraulic moment," tending to open the blades, is Fa , a being $n - m$. The hydraulic moment is balanced by a "reactive moment" Rb , tending to close the blades, produced by a balance piston in the runner hub acting on the blades with lever arm b , R being the total piston load.

An analysis of tests on airfoils similar in shape to turbine blades shows that the distance of the center of pressure (point y in Fig. 1) from the leading edge in proportion to chord length or n/C reaches a minimum value at a moderate angle of attack α , approximately the most efficient angle of attack in the case of a turbine blade. The curve of n/C plotted against angle of attack

is fairly flat for several degrees variation in α from the value giving minimum n/C . But on either side of that value the center of pressure travels downstream, Fig. 2. A further analysis of the characteristics of airfoils shows that the location of the center of pressure varies with the proportional mean camber c/C . This

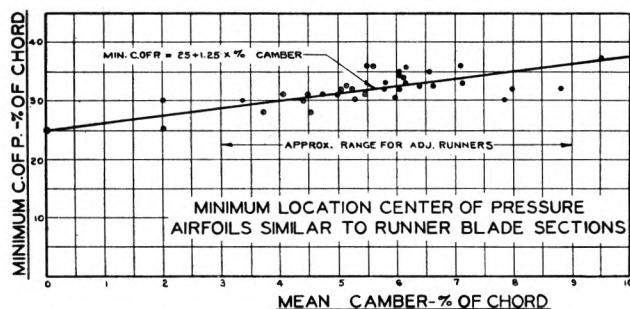
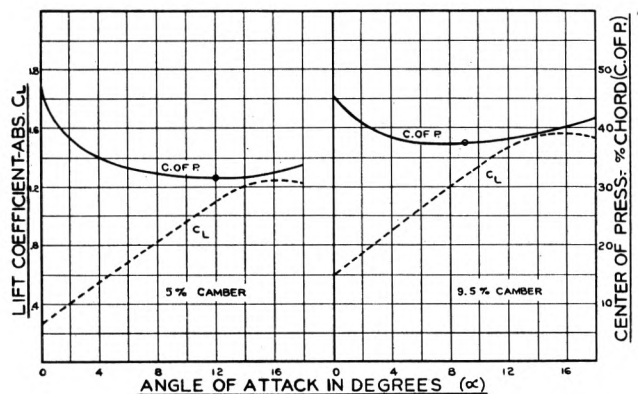


FIG. 2 TYPICAL CHARACTERISTICS OF AIRFOILS

variation has been found to be fairly accurately expressed by the formula

$$n/C = 0.25 + 1.25 \times c/C$$

Thus, a blade section with a mean camber of 8 per cent would have its center of pressure located at 35 per cent chord, and should be pivoted at about 33 per cent chord. The proper camber of turbine blades gradually decreases from the hub to the periphery with a resulting change in the location of the center of pressure. We are primarily interested in the location of the center of pressure of the blade as a whole rather than in its location for each individual section. It has been found more practicable in producing simple blade shapes to place the axis further downstream near the hub and further upstream near the periphery, producing an over-all result as though detail considerations were given to pivoting each section upstream from its individual center of pressure. The runner blade is usually divided into four annular sections of approximately equal discharge and consideration given to the average result.

Fig. 3 shows a typical runner model with six blades, while Fig. 4 shows a sectional view of the model, and Fig. 5 a typical layout of a runner blade.

As may be expected, it was found that blades in echelon, grouped around a common hub, and enclosed in a housing, do not act exactly as they do in the case of single airfoils tested in infinite space. The echelon arrangement, together with the reactive nature of a turbine runner, tends to move the center of pressure somewhat downstream. This tendency has been found to increase with the camber, number of blades, and with blade width

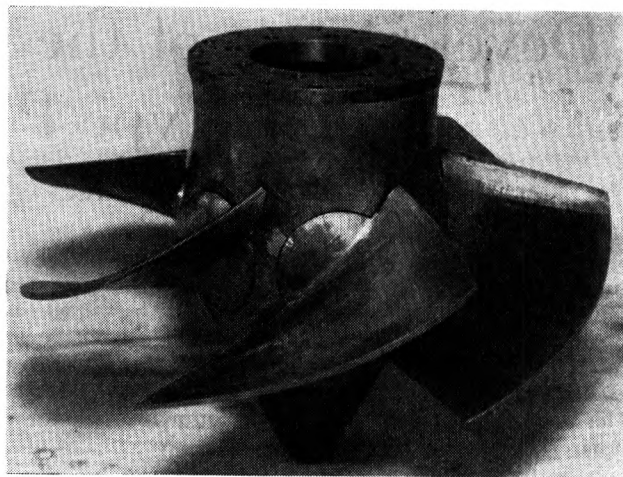


FIG. 3 RUNNER MODEL WITH SIX BLADES; 16 1/2-IN. SIZE

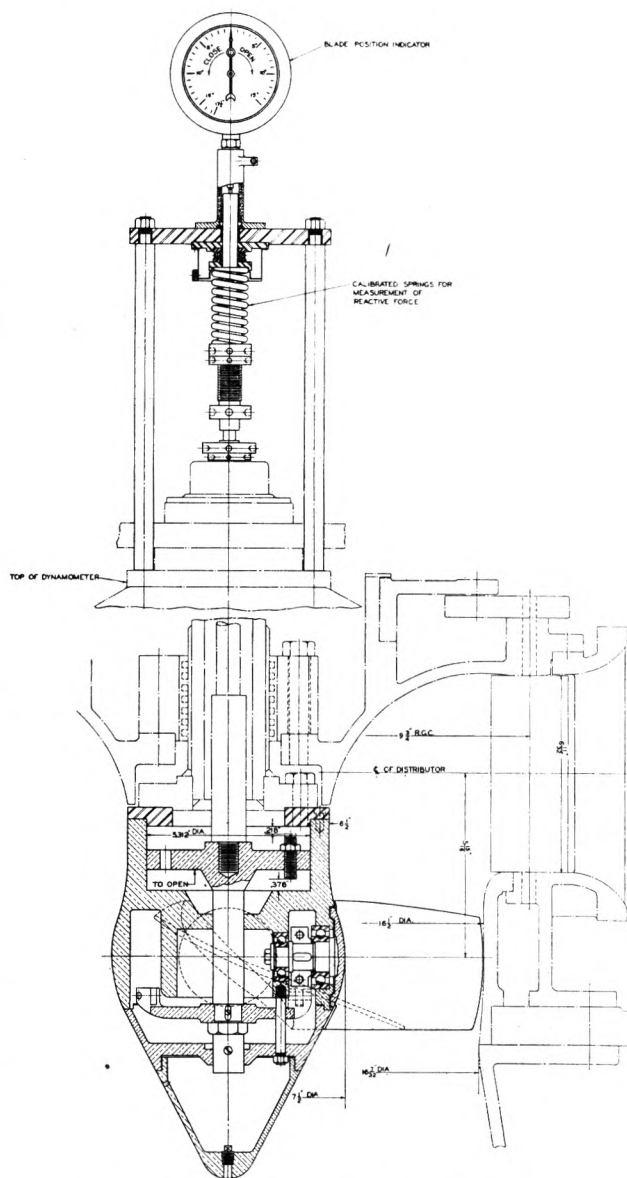


FIG. 4 SECTIONAL VIEW OF 16 1/2-IN. TEST MODEL

or overlap. An idea of the degree of movement of center of pressure may be had from Table 1.

TESTS FOR MODEL DEVELOPMENT

The test procedure for developing a model was as follows:

- 1 Design the blades, based upon data secured from previous tests and from the consideration of the characteristics of airfoils.⁴
- 2 Test the model for speed, power, and efficiency at several blade positions, usually six, with the blades locked.

⁴ "Aerodynamic Characteristics of Airfoils—VI," National Advisory Committee for Aeronautics, Tech. Report No. 315, 1929.

TABLE 1 COMPARISON OF ADJUSTABLE-BLADE RUNNERS FROM TESTS ON MODELS 16½ IN. IN DIAM

Relative blade camber	Normal			Small
Number of blades	4	5	6	6
Type no.	164	165B	166B	179D
Best unit speed, N_1	200	180	160	160
Specific speed, N_s	160	140	120	120
Pivot point, per cent of chord:				
At hub	34	37	40	36
At periphery	24	27	30	21
Average	29	32	35	28.5
Mean camber, per cent of chord:				
At hub	7.7	8.7	8.4	7.1
At periphery	2.3	3.1	3.6	2.5
Center of press, per cent of chord:				
At mean flow line	31.0	34	37.0	30.5
From airfoil tests	30.0	31	32.5	30.5
Discharge angle at periphery, deg.	14.5	16	18.5	18.5
Chord angle at periphery, deg.	17	19.5	22	21
Hub diameter, per cent of outside diameter	40	42.5	45	45
Blade width (plan) ÷ blade pitch:				
Hub	0.88	1.06	1.09	1.09
Periphery	0.77	0.88	1.00	0.91
Blade area, per cent of annular area	95	114	128	117
Head range (approx), ft.	10-40	25-55	40-70	..

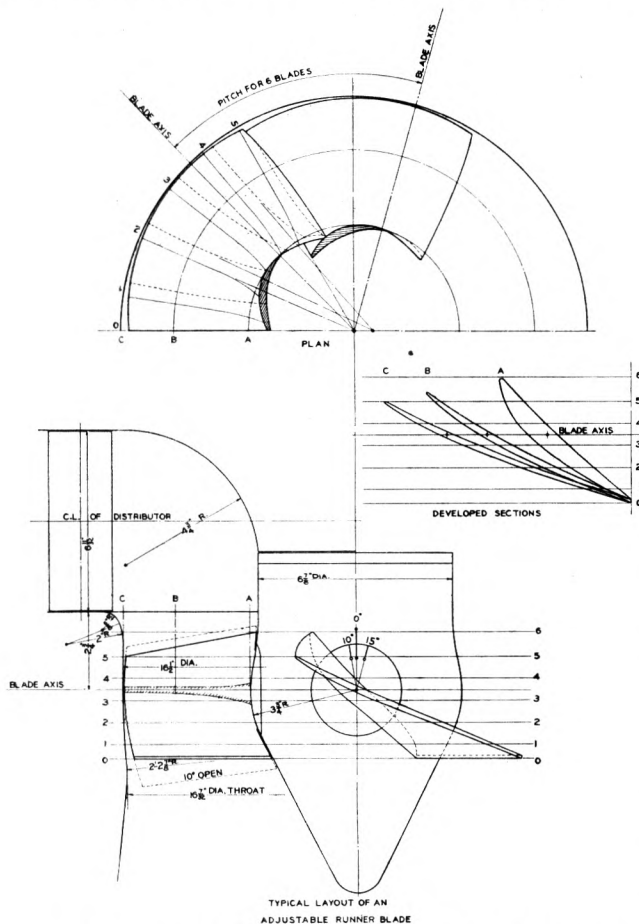


FIG. 5 TYPICAL LAYOUT OF AN ADJUSTABLE RUNNER BLADE

3 Unlock the blades so that they are free to assume a balanced position at different operating conditions. Calibrated springs with various amounts of compression are used to produce reactive moments tending to close the blades.

4 Plot moment diagrams, one for each of several speeds, as illustrated in Fig. 6.

5 Study the required reactive moments for best efficiency operation, best gate-blade relation, for each speed, and reduce them to a constant speed for variable-head operation. In analyzing the moments, gravity effects are also considered.

6 Put the model in automatic operation with a suitable reactive moment with the wicket gates under governor control, driving an electric dynamometer, to determine its behavior under starting and runaway-speed conditions, as well as under normal power-producing conditions.

7 Finally, determine the movement of the center of pressure from the test data. This is done primarily from a consideration of the measured hydraulic moments and from the calculated thrust. However, some blades are tested with two locations of the axis and the center of pressure computed from the two sets of hydraulic-moment data.

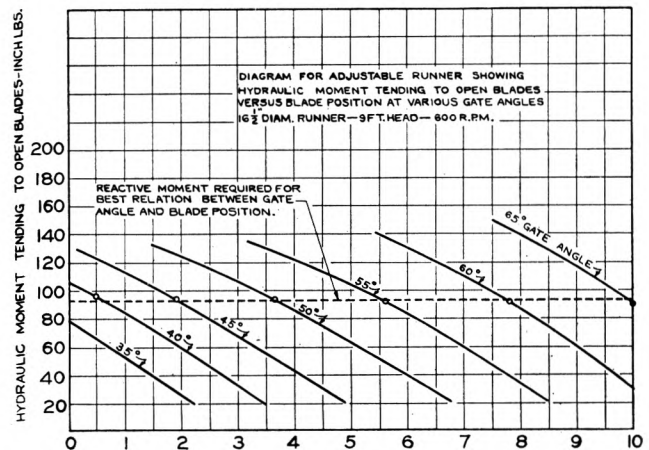


FIG. 6 MOMENT DIAGRAM FROM A MODEL TEST

From Fig. 5, it may be seen that the general shape of blades employed with these adjustable runners is quite simple. A cylindrical section approximates that of a portion of a parabola, with which uniform deceleration of the absolute whirl component of the water is theoretically obtained. The maximum camber is placed at about one third the chord distance from the leading edge of the blade. The face and back of the blades over most of their area are each produced by a system of straight lines (for horizontal sections) meeting at a common point, at the runner axis for the face and near the axis for the back. The blades are laid out for an intermediate angular position, usually for 60 per cent open position. The pitch is made uniform on each radial line. However, the inlet edge of the blade is extended at the hub, resulting in greater pitch at that point. Cambers much greater than those theoretically required from a consideration of velocity diagrams are selected to allow for the bending effect of the water streams before and behind the runner.

As a result of tests to date the following general conclusions may be drawn:

- 1 The blades tend to follow the gates, therefore tend to open as gates open and vice versa.
- 2 With proper blade shape and pivoting, a reasonably constant reactive moment may be used for operation at constant speed at or near best efficiency, for the various gate openings and for varying unit speeds as the head is changed.

3 At low unit speeds, high heads for constant speed, the required reactive moment for best efficiency operation decreases somewhat as the load increases, that is, with a constant reactive moment, the blades move less than required for perfect synchronism with the gates.

4 At high unit speed, low heads, the blades tend to move more than is required for the best gate-blade relation.

5 When properly pivoted, the blades have a strong tendency to open when starting the wheel from rest. This tendency is greatest with 4-blade, and decreases with 5- and 6-blade runners.

6 The blades have a strong tendency to open if the runner runs away at constant head. This effect is least with 4 blades and greatest with 6 blades. The hydraulic moment at runaway speed usually ranged from 4 to 5 times normal with the models tested.

At the present stage of development, it is not considered practicable or even possible to explain all the various characteristics by mathematics and hydrodynamics. The problem is too complicated, considering the numerous variables. However, a discussion of a few concepts is perhaps in order.

1 Why should the reactive moment required for best efficiency be fairly constant for varying loads and heads? To start with, the turbine drives an electric generator at a constant speed. The tangential velocity is relatively high compared with the variable axial velocity. The result is a fairly constant relative velocity w_a , which is the one that produces the predominant dynamic effect on the blades. For best efficiency under the various operating conditions, the angle of attack is naturally about the same, resulting in approximately the same center-of-pressure location. A constant relative velocity, together with a fixed angle of attack and center of pressure, would, of course, create a constant hydraulic moment.

2 Why should the moment increase as the gates are opened? As the gates open, the angle of attack increases. This causes a slight movement of the center of pressure downstream, and an increase in the normal force, resulting in increased moment. This change in conditions overbalances the reactive moment and the blades move open. However, as the blades move open, the angle of attack and normal force decrease and the center of pressure moves upstream until a new balanced condition is reached.

3 Why do the blades open when starting the turbine? With the runner at rest the water from the gates impinges directly against the runner blades at a very large angle of attack. Under this condition the center of pressure is located well downstream with reference to the axis of the blade. Because the blade is stationary and the angle of water deflection large, the force on the blade, per unit volume of water and head, is relatively high. This results in a large opening moment that overbalances the designed reactive moment at a relatively small gate opening. With 4-blade runners, the blade area is considerably less than the annular area between the hub and periphery. The blades of such runners have a very strong tendency to open when starting and the blades will go wide open. With 5-blade runners, and particularly those with 6 blades, the overlap of the blades may be such that the leading edge of one blade obstructs the impingement of water on the next blade. This results in less movement of the center of pressure and a smaller opening moment. Consequently, the blades of such runners do not open as much when starting.

4 Why do the blades open when the wheel runs away? Under this condition two more or less opposing actions take place. One consists of a reduction in the angle of attack which tends to reduce the blade force and moment. The other consists of an increase in the relative velocity, which tends to increase the force and moment approximately as the square of that velocity or speed. The latter action is predominant, resulting in

rapidly increasing moment, particularly at the higher speeds. Here again there is a considerable change in characteristics as the number and overlap of the blades is increased. The opening tendency is stronger with the larger number of blades.

Tests were also made to determine the effect of the shape of the throat ring below the axis on the power and efficiency characteristics as well as on the hydraulic moment on the blades. These included spherical, cylindrical, and intermediate shapes. As was expected, the spherical shape had an effect in moving the center of pressure downstream as the blades approached their open position. It was found better to make the ring cylindrical for 4-blade runners and somewhat curved for the 5- and 6-blade runners. At the higher specific speeds, it was found that, for a given diameter of runner, the reduction in power caused by spherical ring more than offsets any gain in efficiency at the largest blade angles. When efficiency was plotted against power, the curve for a cylindrical ring was slightly above that for a spherical ring near full load. At low and intermediate loads, no difference was found.

The model tests have established a relation between the center of pressure and the axis of the runner blade which should direct attention to the advantages in utilizing the minimum force to move any type of adjustable-blade runner. When the blades are pivoted too far downstream, large moments are produced, particularly at runaway speeds. If the axis of the blade is moved upstream, closer to the center of pressure, the force needed to move the blades is smaller and the machine is safer from the standpoint of overspeed. However, without the use of antifriction bearings, there is a possibility of encountering the combination of blade-and-gate opening for maximum runaway speed and the generators have to be designed for such possibility.

With the new type of adjustable-blade turbine and with the blades properly pivoted ahead of the center of pressure on roller bearings, the theoretical maximum runaway speed cannot conceivably be encountered. The maximum runaway-speed combination occurs at high gate opening with the blades nearly closed and may be from 2.3 to 2.5 times the normal speed, or even higher. With the blades allowed to open fully the runaway speed is held within 1.8 to 2 times normal, the overspeed for which standard generators are designed. Also the force required to move the blades is only a small fraction of that needed when the blades are not appropriately pivoted with respect to the center of pressure.

* INSTALLATIONS

By 1934, it was considered that sufficiently satisfactory results had been obtained in the laboratory to justify an experimental field installation. Arrangements were made at that time with the Kanawha Valley Power Company to furnish a 14-ft 9-in. adjustable runner instead of a fixed-blade runner at the Marmet plant^{5,6} near Charleston, W. Va. This unit was placed in operation in 1935, and has been in continuous use since that time. It is rated 7600 hp under 23 ft head at 90 rpm and has developed about 8500 hp under that head. With the exception of the runner and a few minor differences, the adjustable-blade-runner unit is identical with the adjacent fixed-blade-runner turbine rated 6600 hp. Two other turbines of essentially the same design have since been put into operation for the same company, one in 1937 at the Winfield plant⁷ rated 9150 hp at 26-ft head and one in 1938 at the London plant, both near Charleston, W. Va.

⁵ "Design Features of London and Marmet Hydro Developments," by Philip Sporn and E. L. Peterson, *Power Plant Engineering*, vol. 41, 1937, pp. 80-87.

⁶ "Automatically Adjustable Propeller Turbine," by R. V. Terry, *Power*, vol. 81, 1937, pp. 97-99.

⁷ "Design and Operating Features of the Winfield Hydro Development," by Philip Sporn and E. L. Peterson, *Power Plant Engineering*, Feb., 1940, pp. 36-42, 46.

The Marmet turbine represented a rather large step-up in physical dimensions and power from the model, especially for equipment of such radical departure from past practice. The areas of the water passage were about 115 times and the power about 300 times those of the model. A blade of the Marmet runner is shown in Fig. 7, being machined in a 120-in. lathe. The general location of the blade axis with reference to the blade area is well

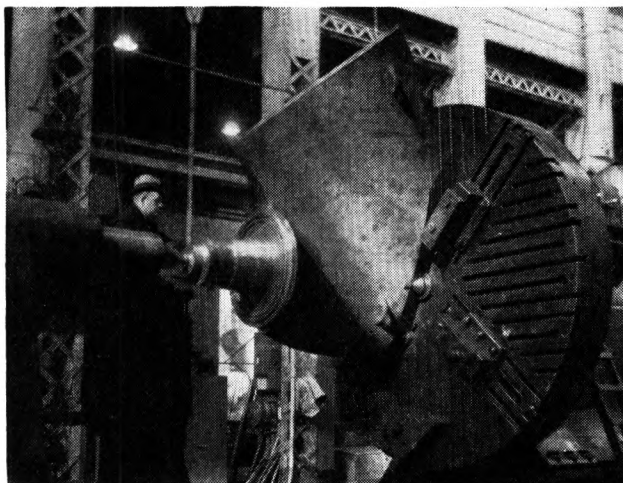


FIG. 7 BLADE FOR 14-FT 9-IN. MARMET RUNNER

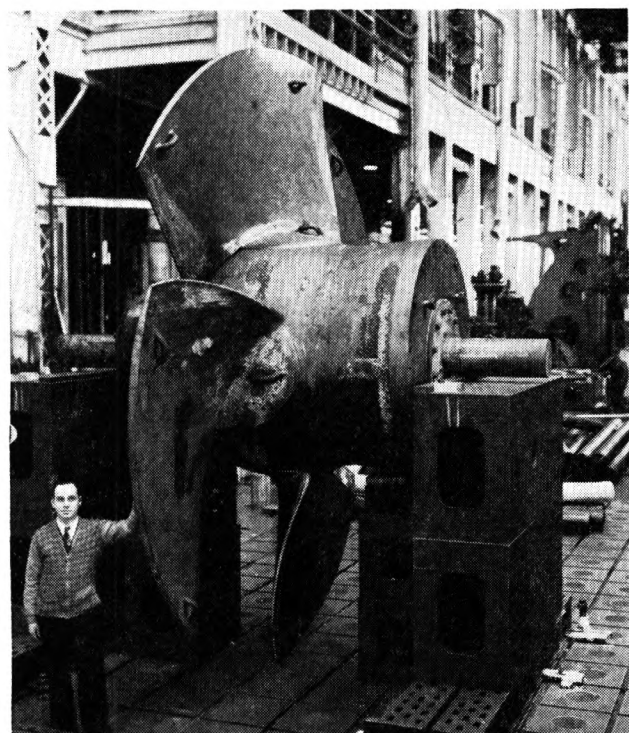


FIG. 8 BALANCING ADJUSTABLE-BLADE RUNNER FOR THE WINFIELD PLANT

illustrated. Another interesting feature illustrated is the relatively large size of the blade boss which is important in obtaining the proper strength of connection between the trunnion and blade. This is made possible by the use of an integral labyrinth seal. Fig. 8 shows the Winfield runner being balanced in the shop and Fig. 9 shows a sectional view of the Winfield unit.

With these three installations, the connection between the four

blades of the runner and the balance piston was of the rack-and-gear sector type, each rack being guided in two bearings. The eight rack bearings cause some friction which creates a lag in the blade position with reference to the wicket gates and results in some differences between increasing and decreasing loads, amounting to about 10 per cent in gate opening or about 3 deg in blade position. These differences are not considered important. Field efficiency tests of those installations were not made because

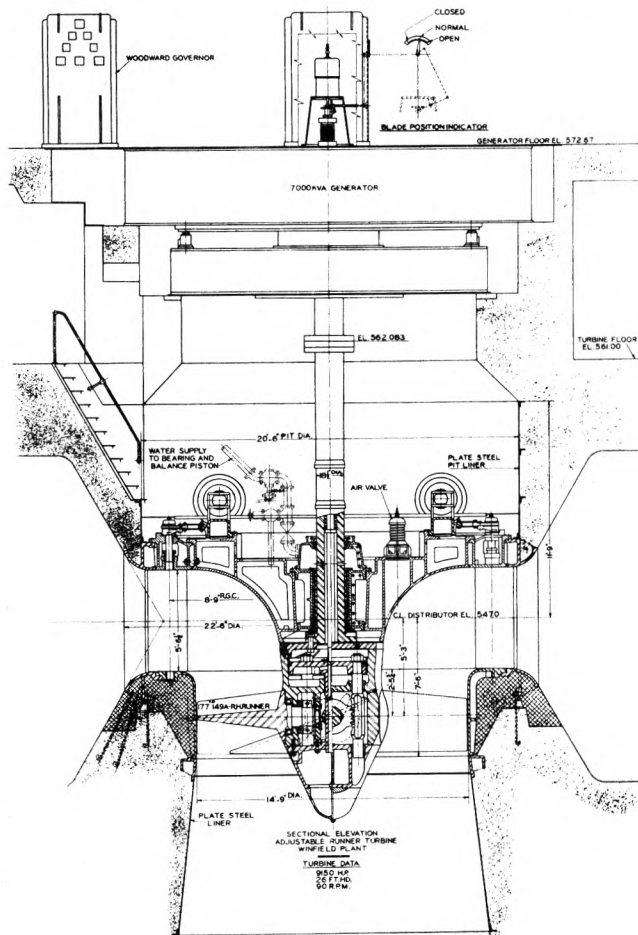


FIG. 9 SECTIONAL VIEW OF 9150-Hp WINFIELD UNIT

of the difficulty of measuring the water accurately. However, the average blade position with respect to gate opening closely follows the best relation determined from the model tests. The power output plotted against gate opening approximates a straight line, which is a desirable characteristic.

The fourth installation was made during the last year at the Austin Dam plant of the Lower Colorado River Authority and consists of two units, each rated 10,000 hp, 200 rpm, 61 ft head. Preliminary field-test results show a maximum efficiency of 92 per cent.

INSTALLATION AT AUSTIN DAM PLANT

This installation may be considered typical and will be briefly described by reference to Fig. 10. The six runner blades are of cast steel with integral trunnions pivoted on roller bearings in a cast-steel hub. Each blade is pivoted on three roller bearings, two radial bearings, and a thrust bearing. The runner hub is divided into two compartments. The lower compartment, which is packed with grease, carries the bearings and blade-connecting mechanism. The upper compartment consists of a bronze-lined

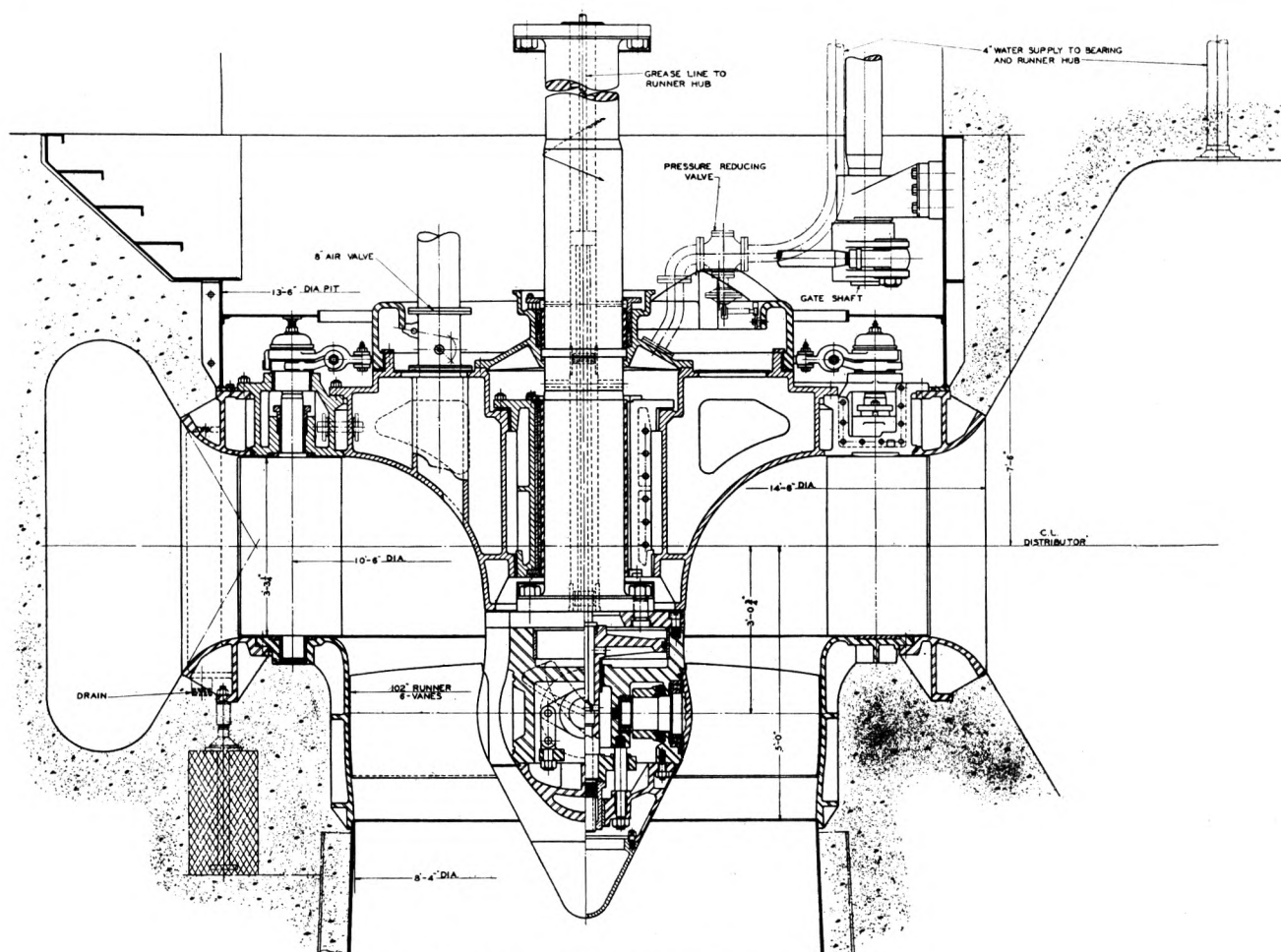


FIG. 10 CROSS SECTION THROUGH AUSTIN DAM TURBINE

cylinder to house the reactive piston, the integral stem of which projects downward through the lower compartment for connection to the blades by spider, links, and levers. The hub is also provided with top and bottom covers of cast steel, the top one being arranged for connection to the shaft. The stroke of the piston and spider limits the travel of the blades to 25 deg. The piston rod is guided in two bronze-bushed bearings made equal in diameter so that the reciprocation of the piston will not create a pumping action on the grease in the lower hub compartment. The space under the balance piston communicates with the draft tube through ports in the piston hub, hollow piston rod, and ports in the lower cap of the runner. Reactive pressure on top of the piston, tending to close the blades, is supplied from the headwater, through strainers and a pressure-reducing valve, to the bearing housing, thence through radial holes in the shaft and downward through the hollow shaft. This supply is combined with the water supply to the rubber bearing. The pressure supply is maintained constant by the pressure-reducing valve, the adjustment of which may be easily altered in the field as desired.

Grease is supplied to the runner hub through a pipe connected internally to the piston rod and extending upward through the generator and turbine shafts. This pipe also actuates the blade position indicator which is mounted above the generator. The top end of the pipe is provided with a swivel connection for greasing the runner hub while the turbine is running.

The runner hub is packed with grease during assembly. A heavy-bodied adhesive grease is used. It is expected that some water will enter the hub. This is, however, made difficult by the

use of the heavy-bodied grease and by the use of a labyrinth-type seal at the blade bosses. The roller bearings are of the 15 to 18 per cent chromium type of corrosion-resisting steel with a Rockwell C hardness of about 55.

In addition to creating the reactive moment, the piston in the upper part of the runner hub also serves as a very effective dashpot. The inflow to and outflow from both sides of the piston must pass through restricted openings. This dashpot action helps to steady the blade movement as well as to limit the rate of movement.

The mechanism in the runners of all installations is simple and rugged. The blades are pivoted only slightly upstream from their normal centers of pressure, approximately 1 per cent of the nominal diameter of the runner. During normal operation this results in rather small moments about the blade axes, and relatively small forces on the internal mechanism, tending to reduce wear. The mechanism is, however, designed to carry forces about 5 times their normal value. This is done to provide for the larger forces which may occur during runaway speed and to take care of a hypothetical condition of having the total moments of all the blades concentrated on the mechanism of one blade.

Externally, all units have the same general appearance as those employing runners with fixed blades. The generators, governors, shaft couplings, and external mechanism of the turbines are of standard construction.

OPERATING EXPERIENCE

The five units now in operation have been in continuous service

since installation of from 6 months to 5 years. No trouble of any consequence has occurred with any of these units. Highly satisfactory operating results have been obtained. All units govern very well. The three Kanawha Valley Power Company units are of the automatic type remotely controlled, and handle well under that type of operation.

As proved by operating experience and as may be seen from a study of the model-blade moment diagram shown in Fig. 6, the blade movement with respect to gate movement is inherently stable. For a given reactive moment and head, there is a rather definite balanced position of the blades for each gate opening. When the gates are moved by the governor, the blades become unbalanced until a corresponding movement of the blades restores the balance. The blades do not overtravel because such a movement sets up a restoring moment. For slow movement of the gates during normal governing, there is enough friction in the blade mechanism to prevent unnecessary movement of the blades. For large changes in load in either direction, the blade movement follows the gate movement rapidly with very little lag.

Perfect synchronism between gate-and-blade position, such as to result in the highest possible efficiency at all loads and heads, is not obtained. However, since blade adjustment is automatic for both head and load, the perfection of adjustment is considered adequate.

When any load up to full load is kicked off the unit, the blades close approximately as fast as the gates. This is a desirable condition of operation because, when the gates are closed, the runner tends to screw up in the water and thus to lift the entire rotor. The blades being in their flat or nearly flat position greatly reduce the lifting effect.

Lifting effect on shutdowns is also reduced by the admission of air through the crown plate. It is customary to provide each unit with two rather large air valves of different types. One is of the check type which is forced open by an adjustable cam as the gates close. That valve takes air through a pipe from the outside of the powerhouse and is also used for venting the turbine during normal operation under load at low gate openings. The other air valve is a spring-loaded check, taking air from the turbine pit, and is adjusted to open at about 15 ft of water vacuum. It opens only when the dropping of load produces a high vacuum under the crown plate.

With the air valves in operation and the runner blades reaching their closed position almost simultaneously with the gates, a highly satisfactory shutdown results. Practically no bump can be felt from the top of the turbine or generator when full load is dropped.

The turbine runners and throat rings have proved to be remarkably free of pitting from cavitation. This rather gratifying experience is attributed to several factors. The models were subjected to extensive cavitation studies. The runner blades are of simple curvature. The position of the gates and the sectional curvature of the top of the throat ring were selected so that the gates do not project over the curved part of the ring when in their full-load position. This tends to reduce inflow vortices and to produce a more uniform velocity distribution.

DEVELOPMENT WORK IN PROGRESS

Additional studies and experimental work are in progress with a view toward further improvement and toward the extension of the upper head range. This also includes additional cavitation studies and further studies toward improvement of mechanical details.

Among the improvements in mechanical details might be mentioned the arrangement, shown in Fig. 11, for the automatic admission of free air to the draft tube at the runner cap. This is for

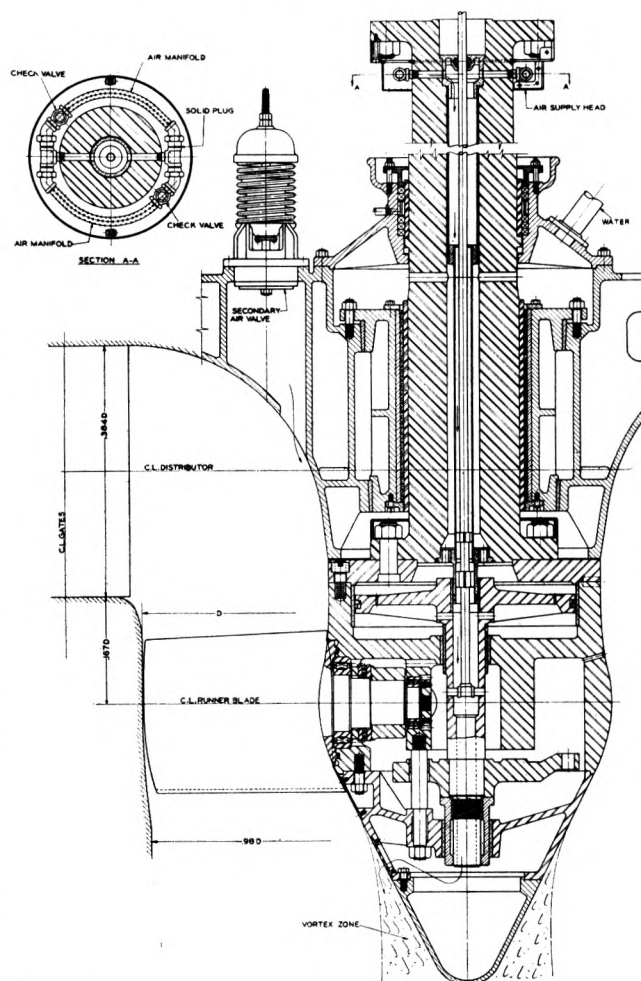


FIG. 11 SECTIONAL VIEW SHOWING ARRANGEMENT FOR AUTOMATIC ADMISSION OF FREE AIR TO DRAFT TUBE

the purpose of giving smoother operation in cases where the water leaves the runner with a considerable whirl. Such an arrangement is advantageous in preventing vibration in the case of runaway speed or operation at greatly reduced heads, high values of unit speed. Whenever a vortex forms at the runner cone in the center of the draft tube, a vacuum is produced, even with high tail water, such as to suck in free air through the tubular passages in the piston rod and turbine shaft. These passages terminate with check valves near the upper end of the turbine shaft. The check valves are adjusted so that they will admit air to the draft tube only when a strong vortex occurs at the runner cone.

CONCLUSION

In conclusion, a summary of the principal features of the new type of hydraulic turbine may be given as follows:

- 1 Blade-operating mechanism is entirely separate from the governor oil-pressure system and is controlled by the turbine water supply. No extra governor equipment is required and all oil pipes are eliminated.
- 2 Blade-adjusting mechanism is concentrated in the turbine proper, reducing interdependence of turbine, generator, and governor design.
- 3 Turbine and generator shaft couplings are of standard construction, the same as for fixed-blade turbines.

4 For normal blade adjustment, forces and moments required to actuate the blades are a minimum due to location of pivot point and use of roller bearings.

5 Blades open when starting turbine, tending to protect the main thrust bearing.

6 Blades open at runaway speed, appreciably reducing overspeed and cost of generator.

7 Use of heavy grease in hub versus heavy oil.

8 Inherent tendency for blades to adjust themselves to the condition of operation.

9 Blades follow gates rapidly on quick shutdowns, greatly reducing lifting effect and forces on the blades.

10 Provision for automatic suppression of vibration with free air in the case of overspeed.

Discussion

C. S. ADAMS.⁸ The information contained in this splendid paper is evidence in itself of the great contribution that has been made to the progress of hydraulic-turbine design by the author. The amazing ingenuity of the inventive engineer should not be permitted, however, to overshadow the tremendous amount of mental and physical energy which has been expended by this man and his associates throughout the last 10 years in order to develop and perfect the original conception of the Newport News type of automatic adjustable-blade propeller turbine.

It was the writer's opportunity and pleasure, when serving as designing engineer for the Lower Colorado River Authority under Clarence McDonough, general manager and chief engineer for the Authority, to have been permitted to study the development of this turbine through the laboratory stage; to have viewed the first commercial installations on the Kanawha River in West Virginia; to have designed the Austin hydroelectric power plant around the two most powerful of these turbines yet built; and to have installed these two turbines and to have placed them into successful operation. Through each of these

stages of development and application, the hydraulic engineer can find that this water-operated automatic adjustable-blade turbine has definitely proved a successful reality.

The writer will endeavor to sketch briefly some of the items of special interest which concern the designer, constructor, and operator of power stations that embody this new type of turbine as well as to present a few brief facts relative to the Austin installation.

The invention of the Newport News type of adjustable-blade-propeller turbine has furnished the designing hydroelectric engineer with another useful tool which can be applied advantageously to assist in the solution of problems that occur in the design of hydroelectric power systems where the conservation and the efficient use of water in the system must be effected to the utmost. The Newport News turbine permits a simple, neat, compact, and relatively inexpensive installation to be provided at hydraulically proper locations in order to supply energy and power to small "off-peak" loads with a relatively high over-all plant efficiency.

To cite a practical application, each of the two Austin turbines is rated at 10,000 bhp at 200 rpm under a net head of 61 feet. This 20,000 bhp of adjustable-blade propeller turbine is an effective part of the Lower Colorado River Authority's total of 175,000 bhp which is now installed in four hydroelectric power plants on the Colorado River of Texas. In the ultimate design of this system, it was considered necessary to install the 20,000 bhp of adjustable-blade turbines at Austin so that the 155,000 bhp of Francis turbines along with over 2,000,000 acre-ft of firm water storage could be so correlated and coordinated that the over-all operating efficiency of the entire system under commercial loads could be at the highest possible value. A thorough study of the entire project with respect to hydrology, reservoir characteristics, and commercial power sales disclosed the definite necessity for an adjustable-blade installation.

The physical conditions at the Austin site rendered the installation of power machinery difficult. An unusually high tail-water condition during floods and the long length of spillway required for the dam in order to accommodate flood flows, coupled with the fact that the entire power plant had to be placed in

⁸ Frederic R. Harris, Inc., Consulting Engineers, New York, N. Y.; formerly Designing Engineer, Lower Colorado River Authority, Austin, Tex.

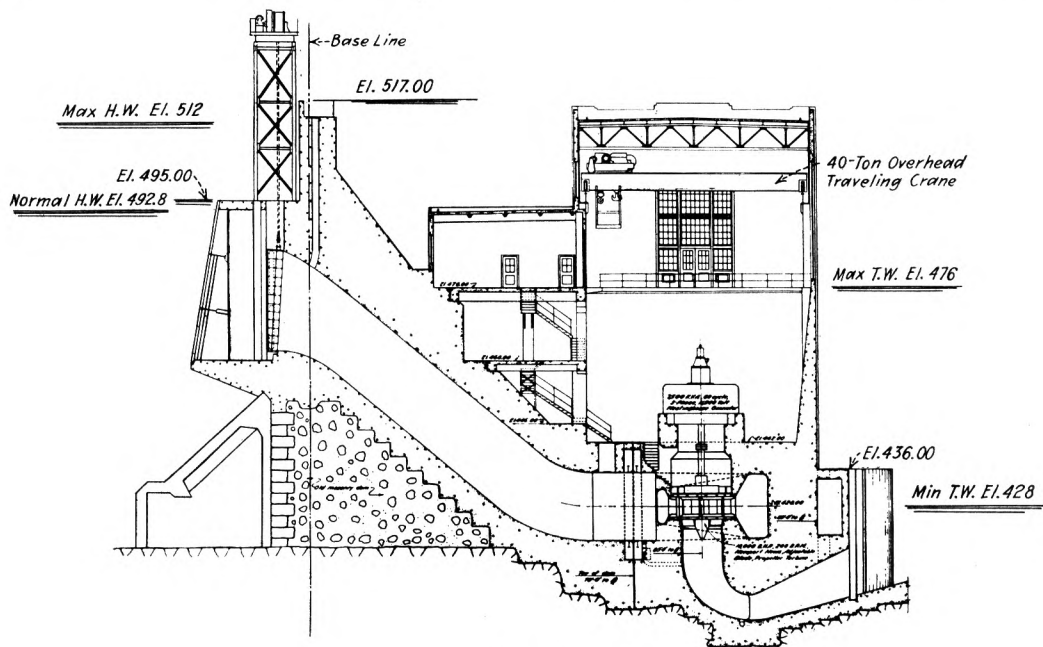


FIG. 12

the channel of this major stream, made necessary the most compact installation that could be developed. Figs. 12 and 13 of this discussion, showing the transverse and longitudinal cross-sections through the power station will enable this compactness to be observed.

The Newport News turbines lent themselves admirably to a neat and compact setting. An important factor in this particular power-plant design, in so far as the entire system was concerned, was that the demand capacity obtainable with the given diameter of runner was at a maximum with the adjustable-blade type. The design also permitted the use of 60,000-lb torsional gate-shaft Woodward hydraulic governors in a pleasant-appearing cabinet form, as may be seen in Figs. 14 and 15 of the interior of the powerhouse, and a minimum of space was required on the generator-room floor and in the turbine pits for the governing installation.

The model tests on the Newport News runner revealed that both a lesser runaway speed and a lower starting hydraulic thrust existed on this type of runner than on the Kaplan type of corresponding characteristics and size. These features resulted in a saving in the electric generators together with an improvement in operating conditions. The author in pivoting each of the runner vanes slightly upstream from its center of pressure, has permitted these favorable characteristics to become innate properties of this turbine.

The incorporation of these turbines into the detailed design of the Austin power station entailed but slight additional difficulties, as compared with the Francis type or the fixed-blade-propeller type of turbines. The provision for the supply of water to the blade-actuating mechanism is essentially the only additional service that must be supplied. The governors and generators are essentially the same as they would be for fixed-blade units, although a hollow generator shaft is required for operating the blade position-indicating device atop the direct-connected exciters and for the admission of grease into the hub of the runner.

The installation of the Austin hydraulic-turbine units proceeded without delay. The assembly and, consequently, the dismantling of the units is a relatively simple operation and, since the runners were completely assembled in the shop prior to the field installation, a minimum amount of field-assembly work was required. No more working space in the powerhouse was required for erecting or dismantling these complete generating units than for comparable fixed-blade units. The governors were

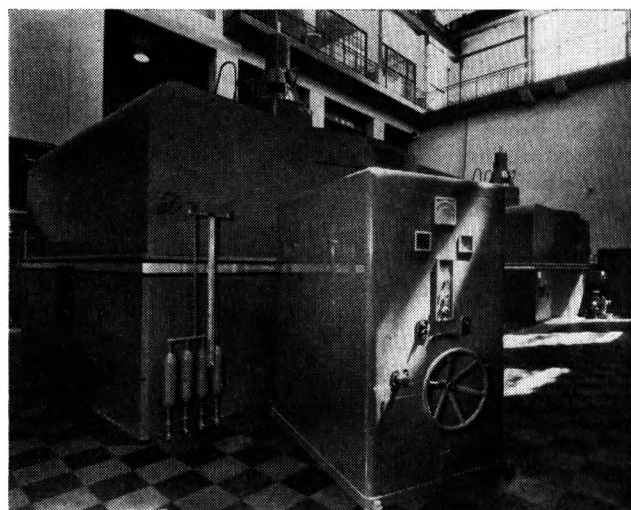


FIG. 14 INTERIOR VIEW OF AUSTIN POWERHOUSE, SHOWING TORSIONAL GATE-SHAFT HYDRAULIC GOVERNOR CABINETS IN FOREGROUND

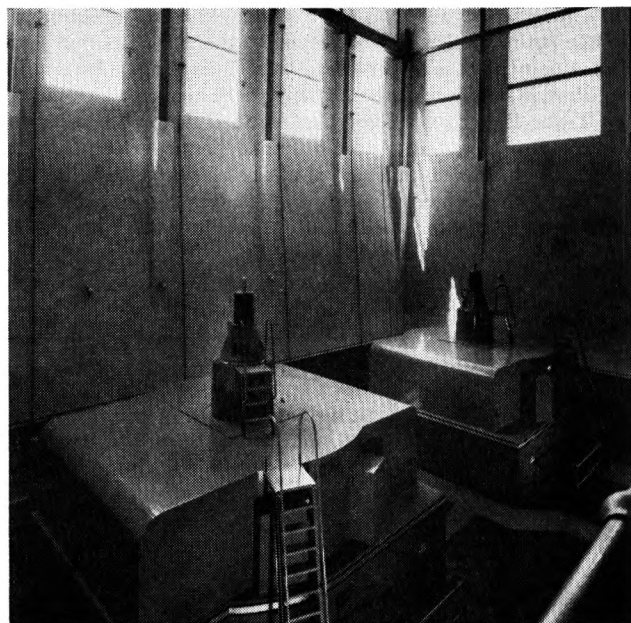


FIG. 15 ANOTHER VIEW IN THE POWER PLANT SHOWING THE TWO 10,000-BHP HYDRAULIC TURBINES

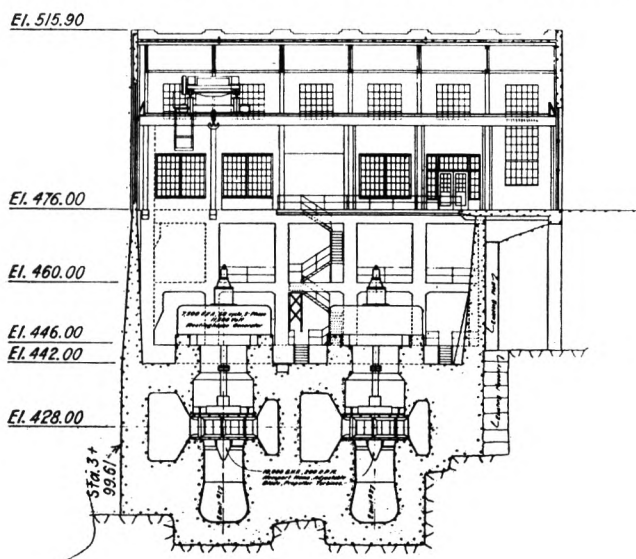


FIG. 13

shipped completely assembled in their housings, and it was only necessary to set them upon their anchor bolts and connect the servomotor connecting rods to the torsional gate shafts.

After the two units had been installed, a short operating period for each unit at about one-half rated speed enabled the Kingsbury thrust and guide bearings of the umbrella type on the Westinghouse generators and the water-lubricated rubber guide bearings of the turbines to be "run in" and to determine that the units were physically completed and also properly installed and serviced. At the end of this "run-in" period, each of the units was loaded with a water rheostat in order that it might be put through a vigorous and thorough mechanical testing before being placed into commercial operation, and in order that the operation of the runners could be observed and adjusted to insure performance characteristics comparable with those of the final laboratory model.

Load was applied to each generating unit in increments of about 1000 bhp; the unit was operated for about 5 min, then the circuit breaker was opened to remove the load; and the governor closed the regulating gates at the rate of 3 sec for a full gate stroke. As the units were started up after each run and as loads were increased to a maximum of 5 per cent above the rated capacity, it could be noticed that the blades opened almost to the fully open position, just as the regulating or wicket gates began to open, but the blades then closed slowly to the proper position. When the unit was disconnected from the bus, the blades closed at almost the same instant that the 3-sec governor closed the regulating gates. The quick-opening characteristic lessened the hydraulic thrust on the main bearing caused by the inrush of water on starting the unit, whereas the quick-closing characteristic denoted the sensitivity and fine mechanical balance of the individual vane bearings and linkage mechanisms for each of the six runner vanes of the 102-in-diam runner.

After the thorough initiation and adjustment with the aid of the water rheostat, the two units were placed in commercial operation on April 1, 1940. Although the flywheel effect or WR^2 of the Austin generator and turbine rotors is small and the flywheel effect of the rotating machinery of the connected loads is not great, the generating units governed perfectly. The units governed so well that the frequency-controlling equipment for the entire system will be moved to the Austin power plant. The Austin plant also serves as the dispatching point for the distribution of power to the Authority's customers which include the Texas Power and Light Company, Houston Lighting and Power Company, San Antonio Public Service Company, the City of Austin, 3500 miles of rural-electrification lines, and the Authority's own power district which is approximately the same area as the combined states of Massachusetts, Connecticut, Rhode Island, and New Jersey.

These two units have now been operating satisfactorily and continuously for 8 months. Outside of an occasional replacement of breaking pins on the wicket-gate mechanism, the units have functioned perfectly and on occasion have operated continuously at full rated load for a period of two weeks.

In July, 1940, each of the turbines was given a thorough preliminary field test in order to determine its performance characteristics. The Gibson method was used for determining the quantity of water used at each test point and all observations and computations were made essentially in conformance with the A.S.M.E. Power Test Code for Hydraulic Prime Movers. The units were tested between 9 a.m. and 8 p.m. with commercial loads. A reasonably flat horsepower-efficiency curve was obtained for each unit, but the author considered that the characteristic curves could be improved to conform more closely with the model curves by making a minor alteration to the pressure-regulating device which actuates the servomotor piston of the vane-position mechanism. This minor alteration has now been made and the load versus blade-angle relationships for each Austin turbine corresponds with those of the homologous adjustable-blade model over the entire range of load. The turbines will be given a final test in the near future in order to determine their performance curves.

The maximum over-all efficiency of one turbine, as determined from the preliminary field testing, occurred at 85 per cent of the rated capacity at a value of 92 per cent, whereas, the other turbine attained a peak efficiency of 90 per cent at 85 per cent of the rated capacity. The mechanical operation, the balance of the units, the functioning of bearings, and the operation of the complete generating units, as revealed during the field tests are all considered at or even above par with the other modern hydraulic-turbine installations at the Authority's power plants.

With the development, success, and the experience gained by

the author in the design, manufacture, installation, and operation of the existing automatic adjustable-blade turbines, the hydraulic-turbine field should now be clear for the production of larger-capacity wheels of the same type. It is anticipated, moreover, that the talents and energies which have been utilized in the development of this successful invention will enable further advances to be made in the hydraulic-turbine as well as in related fields.

F. NAGLER.⁹ The author deserves a great deal of credit, as much for initiating a new policy of engineering presentation as for the extremely interesting type of turbine development indicated. In past years, few engineers were either willing or free enough from commercial considerations to be entirely open with engineering uncertainties, field troubles, and the like, to permit that frankness on which the most rapid progress may be realized.

The magnitude of the actual installations described in this paper is particularly noteworthy. The writer and J. F. Roberts did some work on automatically adjustable blades during the early '20's. This work attempted to get around the influence of variable friction, by hanging the blades on knife-edges instead of letting them revolve in the bearings. The idea was that they would be positioned by a combination of centrifugal force and water flow. The work was discontinued because of assumed insurmountable obstacles connected with weakness of structure and the wide variables introduced by changing friction, by variable-velocity conditions as the head changed, and, of course, by the variable velocity and direction imposed by guide-vane operation. The author, apparently, has worked out a feasible solution of these major difficulties.

It would be appreciated if the author would comment a little further on the life of the antifriction bearings on the runner-blade pivots. We know that tremendous forces exist on these blades; forces, as pointed out in a paper¹⁰ by J. D. Scoville, of a magnitude sufficient to lift the entire rotor of generator and turbine. These forces come practically as a blow, Fig. 9¹¹ in the Scoville paper.

In a current paper¹² H. Styri brought out the point that, to insure commercial life of any ball or roller bearing, there must be an oil film present between roller and race. It is probably not particularly difficult to maintain such a film for a bearing that rotates, but the bearings in the hub of the runner are practically stationary throughout their life. The writer would expect certainty of breakdown of the oil film and inevitable peening action under such conditions. Undoubtedly, the field experience to date is the most effective answer to a question as to life of antifriction bearings under this duty. Further comment from the author on this point would be appreciated.

When a piston interconnecting and influencing the position of the blades is added, as shown in Figs. 9, 10, and 11 of the paper, and external control of the pressure behind that piston taken outside the shaft, are we not approaching closely an externally adjustable blade, the connection of which to the guide-vane motion would require a relatively minor addition in the form of links or cams?

It would be exceedingly interesting, if the author would show a comparison between the shape of the efficiency curves of the automatically adjustable-blade construction, and the Kaplan and the fixed-blade types.

⁹ Chief Engineer, Canadian Allis-Chalmers, Ltd., Toronto, Canada. Life Member A.S.M.E.

¹⁰ "Speed Regulation of Kaplan Turbines," by J. D. Scoville, *Trans. A.S.M.E.*, vol. 63, 1941, pp. 385-394.

¹¹ *Ibid.*, Fig. 9, p. 389.

¹² "Friction Torque in Ball and Roller Bearings," by H. Styri, *Mechanical Engineering*, vol. 62, no. 12, December, 1940, p. 886.

The use of air valves, as applied to this type of construction, is of particular interest to the writer since, at the time of presentation of this device,¹³ but slight attention was paid to its utility. Several major field accidents would have been prevented had this device been adopted more generally. This is particularly true on the axial-flow types of turbines, where such positive uplifts as are reported by Scoville¹⁰ are encountered. There are some instances reported of uplift with the Francis turbines, but the writer has not personally observed them. It would be of interest if the author would comment on this particular feature.

J. F. ROBERTS.¹⁴ The description in the paper of the various tests and model and field experiments which have gone into developing the automatic adjustable-blade-type propeller turbine is both interesting and enlightening. The results should be very gratifying to the author and should result in a real saving to the users of this type of turbine.

The author's remarks concerning the use of air valves are interesting. The writer has always been a strong advocate of ample size of air valves, both for fixed- and adjustable-type propeller turbines and for Francis turbines. The point where the air is admitted to propeller-type runners has been found quite important and, in several cases, a change in the point of air admission has resulted in a material improvement. The writer has found that, if the air is admitted to a propeller turbine at the lowest point in the head cover, just above the top of the runner hub, the greatest benefit can be secured. In two cases it was found possible to get air into the turbine at the lower point when it would not take in air near the upper part of the head cover, due to a positive pressure existing at that point.

It has also been found unnecessary to use a cam or linkage to open the air valve at definite gate openings and close it at other gate openings since, both on Francis- and propeller-type turbines, the air is apparently beneficial whenever a vacuum exists which is sufficient to draw the air into the turbine. By providing the air valve with a light spring-loaded check valve so that it will close at zero pressure and prevent water from flowing out of the valve, but will open whenever a vacuum of 1 or 2 in. of mercury exists, the use of cams connected to the gate mechanism can be eliminated and better results obtained under both varying heads and varying gate openings.

J. D. SCOVILLE.¹⁵ The author's turbine is called an automatic adjustable-blade propeller, the blades moving to a new position as the gate opening is changed because of inherent characteristics of the runner. This is distinguished from the ordinary Kaplan turbine in which the blades are moved by an oil-operated servomotor, controlled by the gates. This might be called a controllable-pitch propeller.

There are certain differences between the two types which should be pointed out. The author states that, when the gates are opened on the automatic adjustable-blade turbine, the blades go to a large angle which make starting easy. On the controllable-pitch propeller, the blades are normally in their flattest position on starting, so that one would expect a large gate-opening requirement to start the unit. Such is not the case. Usually 10 to 15 per cent is sufficient. The unit reaches synchronous speed at about the same opening.

As the gates open further on the automatic adjustable-blade

propeller, the blades follow. When the gates open on the controllable-pitch propeller, a cam on the gate shaft or connected to the gate servomotor controls the blade position. The shape of this cam is determined by index tests in the field so that the blades are moved positively to the best position. As the head changes, a new blade-gate relationship is required. This alteration can be taken care of by an adjustment in the cam position which requires only a minute or so, the turbine remaining in normal operation.

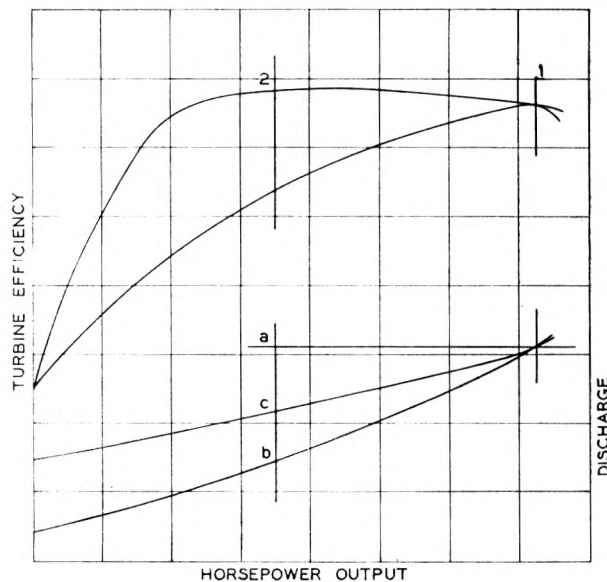


FIG. 16 CURVES SHOWING RELATION BETWEEN LOAD CHANGES AND DISCHARGE OF CONTROLLABLE-PITCH PROPELLER

The blades of a controllable-pitch propeller can be moved as fast as the gates for load changes. It is desirable, however, to restrict the closing of the blades. The reason will be evident from Fig. 16 of this discussion. If the load changes from 1 to 2 the discharge changes from *a* to *c*, if the blade-closing time is very slow, but from *a* to *d* if the blades close as fast as the gates. Therefore, the pressure change is correspondingly less. This is an advantage in a plant with a relatively long penstock. Usually the controllable-pitch propeller is so adjusted that the blades close in about 60 sec, so that, during a load reduction, it is in effect a fixed-blade runner.

If the controllable-pitch propeller runs away, the maximum theoretical speed will be reached with the blades in a relatively flat position and the gates wide open. It is only possible to obtain this condition if the blades are deliberately blocked at the flat angle. Even if the oil pressure fails and the gates open for some reason, the blades will open because of the unbalance, which the author points out. The opening tendency of the blades on the Bonneville units at runaway speed was about 4 times the force required to open the blades at normal speed. This means that the runaway speed of the controllable-pitch propeller cannot reach the theoretical maximum.

R. E. B. SHARP.¹⁶ The Terry automatic adjustable-blade-propeller turbine presents a definite and valuable contribution to turbine design and to date turbines of this type have given a satisfactory account of themselves. The study and experiments, which the author has made on the hydraulic moments acting on

¹³ "Operation of Hydro-Electric Units for Maximum Kilowatt Hours (The Turbovent)," by F. Nagler, *Engineers and Engineering*, vol. 42, 1925, pp. 148-156.

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¹⁵ Assistant Chief Engineer, S. Morgan Smith Company, York, Pa. Mem. A.S.M.E.

¹⁶ Chief Engineer, I. P. Morris Department, Baldwin Southwark Division of The Baldwin Locomotive Works, Philadelphia, Pa. Mem. A.S.M.E.

runner blades, represent a valuable contribution in connection with the design of the adjustable-blade-propeller turbine, whether of the Terry type or of the usual Kaplan adjustable-blade-propeller turbine.

There is one factor, however, which the author does not mention, but which does have an effect on the net hydraulic moment acting on runner blades, and that is the centrifugal force acting on the blades.

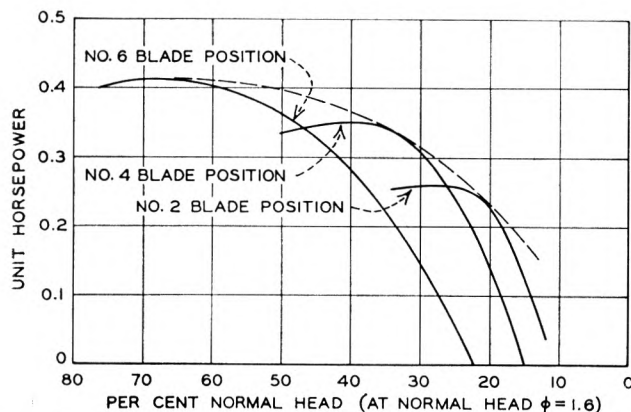


FIG. 17 RELATIVE POWER DEVELOPED BY KAPLAN AND TERRY DESIGNS AT LESS THAN NORMAL HEAD

Referring to the plan view of the author's Fig. 5, all particles in the blades are acted upon by centrifugal force which has a component tending to flatten them. The moment caused by this component is of course greatest at runaway speed. However, the net force mentioned by the author, namely, that resulting from a reduction in the angle of attack and that resulting from the increase in the relative velocity, increases at a greater rate than the centrifugal-force moment. The result is that, with this design of blade, the net moment is greatly increased, tending to open the blades.

When adjustable-blade-propeller turbines are operated at less than normal heads, the Kaplan type of turbine, with the blade-gate relation positively established by a cam, develops greater power than with the author's design; Fig. 17 of this discussion indicates this condition.

The curve marked No. 6 blade position represents the maximum power that can be developed in the steepest blade position for this particular runner. Similarly the curves marked No. 4 and No. 2 blade positions represent the maximum power which can be developed with the blades set in these flatter positions. It is noted that at 30 per cent of the normal head, for instance, the unit power developed for the No. 6 blade position is 0.15, whereas, for a flatter blade position, as read from the envelope, the maximum unit power is 0.315. This condition approaches the runaway-speed condition where the author's blades tend to take up their steepest blade position or angle. In order, however, to develop the maximum power possible from the unit, it is seen that the blades should be definitely brought to a flatter position.

It is the writer's experience, based on comparative tests, that a better performance as regards cavitation and maximum output obtainable are secured by having the blade axis located farther downstream than is indicated in the paper. This is due to reduced clearance between the throat ring and the runner blades in the vicinity of the trailing edge being secured with the axis so located. While this does increase the capacity required of the blade servomotor, it is considered desirable.

The greatest factor in determining the capacity of runner-blade servomotors of Kaplan turbines is the friction in the runner-blade

mechanism in the hub. A minimum amount of friction is therefore quite desirable on that type of turbine and the use of anti-friction bearings in Kaplan hubs would of course accomplish this result to a very great degree. The inevitable entrance of water into the hub at some time or other, however, dictates that these bearings should be of the stainless-steel type, such as the author employs. The slow oscillating movement of the runner blades with a turn of not more than 30 deg, with indefinite more or less stationary periods in conjunction with the live character of the load, tends to cause Brinelling or local strain hardening or grooving action on the races which, in the writer's opinion, has made the use of this type of bearing questionable, particularly in view of the present inability of bearing manufacturers to obtain a satisfactory stainless antifriction-bearing material.

The author has naturally employed bearings of ample proportions and liberal ratings. However, the writer would be interested in knowing whether any of these bearings has been examined after extended field use and whether the lag mentioned in connection with the Winfield turbines can be ascribed to possible grooving of the races. The continued absence of friction is of course essential to the Terry design as a friction increase aggravates the lag between the exact position desired and that actually secured.

One of the interesting features of this turbine, as the author points out, is the tendency of the blades to open to their steepest angle at runaway speed, with consequent reduction in the runaway speed reached, as compared with what would be reached with a flat blade angle. Under normal conditions, the blades both of the Terry turbine and the conventional Kaplan turbine move to their steepest blade positions if the gates open wide under runaway-speed conditions.

It is a common practice among Kaplan designers to assume that the cam relation might be destroyed and the runner blades might conceivably be in the flat position with the maximum gate opening causing high runaway speed. For this to happen however a very remote chain of circumstances would have to exist. Nevertheless, it is considered sound engineering to assume that they might exist.

It appears to the writer that the mechanism of the Terry turbine might prevent the blades from taking up their steepest position during runaway, due possibly to the jamming of one of the blade links or other parts of the hub mechanism, or the malfunctioning of the pressure-reducing valve, controlling the servomotor pressure. Therefore, with the Terry turbine no less than with the Kaplan, it appears desirable to allow for the maximum possible runaway speed with the flat blade position, in view of the possible destructive results should this condition not be provided for.

AUTHOR'S CLOSURE

Several of the discussers raised questions regarding the power-efficiency characteristics of this type of turbine. The Austin dam units were field-tested in January, 1941, according to the standards of the 1938 A.S.M.E. Test Code for Hydraulic Prime Movers. The discharge was measured by the Gibson method. The results of these tests on one unit are shown in Fig. 18 of this closure, in comparison with the results of tests made at several blade positions on a 16 1/2-in. model under 12 ft head in the hydraulic laboratory of the author's company. It will be noted that the efficiency curve is quite flat, being above 90 per cent for about 62 per cent of the load range. The maximum efficiency obtained was 93 per cent. The average efficiency from the 20 individual test points, within the guaranteed range of from 4000 to 10,000 hp, was 91.6 per cent. The author believes that these test results are at least equal to any that have been obtained with turbines of the adjustable-blade type.

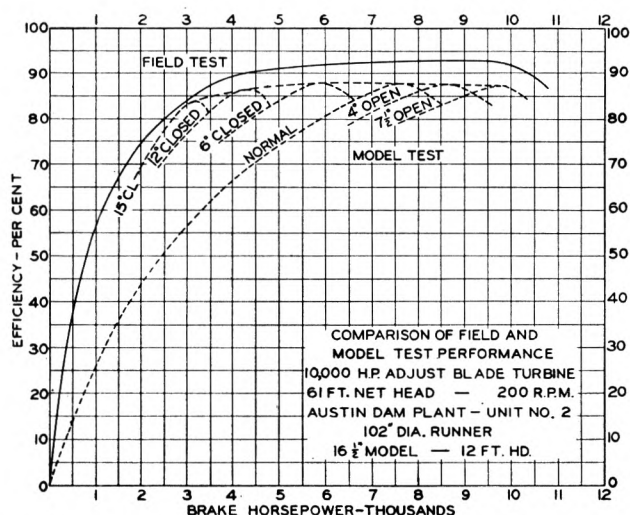


FIG. 18 COMPARISON OF FIELD AND MODEL-TEST PERFORMANCE OF 10,000-Hp ADJUSTABLE-BLADE TURBINE

Mr. Adams' discussion, relating to the installation and operation of the Austin turbines is gratifying, particularly that part dealing with governing and system-frequency control.

Messrs. Nagler and Sharp raise questions in regard to the service and life of the stainless-steel roller bearings employed in the runner hub. The estimated loads, the bearing manufacturer's ratings, and other data on the several bearings of the Austin runner are given in Table 2.

TABLE 2 AUSTIN RUNNER ROLLER BEARINGS

	Outer radial	Inner radial	Thrust
Bore of bearing, in.....	9.00	5.00	8
Outside diameter of bearing, in..	14.50	8.25	12
Bearing length, in.....	2.75	2.19	2
Diameter and length of rollers, in.	$1\frac{1}{8} \times \frac{1}{4}$	$\frac{3}{4} \times \frac{1}{2}$	$\frac{3}{4} \times \frac{1}{8}$
Number of rollers.....	26	21	26
Estimated load, lb.....	81,000	51,000	30,500 (Normal speed)
Rated load (2 rpm), lb.....	109,500	55,400	122,000 (Runaway speed)

The rated loads are based on a speed of 2 rpm, whereas, the actual service approaches a static condition under which the ratings would be somewhat increased. The ratings are also based upon a specified minimum of 52 Rockwell C hardness, whereas, the actual hardness is about 55 Rockwell C. That change in hardness increases their actual capacity from 70 to about 90 per cent of the capacity of carbon-steel bearings, an increase of about 28 per cent.

The life of the bearings for this service is not definitely known. Although the aggregate service of the five units installed is about 15 years, there has been no occasion to dismantle a runner for an examination of the internal parts. If the life of the bearings should prove less than expected, bearings of 25 to 30 per cent higher capacity are now available, interchangeable with the present bearings. That change in capacity would give a theoretical increase of life of from 50 to 75 per cent. A still greater factor of safety or longer bearing life may be obtained by the use of "quill-type" inner bearings or by employing a slightly larger diameter of runner hub. To date, there has been no indication pointing toward bearing failures.

The seventy-two bearings used in the five units in operation were all supplied by one bearing manufacturer who has co-operated in making this new application practicable. The materials used are of a commercial quality which appears to be quite satisfactory. The service required of the bearings is, in a way, somewhat crude as compared with that ordinarily required

of bearings on high-speed machinery. It is the author's opinion that the bearings for this service will prove satisfactory for a much longer period, proportionately, than with most applications of antifriction bearings.

It would appear that roller bearings are inherently more suitable for adjustable-runner application than are the plain bronze-bushed bearings. Since the axial length of the roller bearings is short, the loads are uniformly applied and the unit loads may be readily computed. With the much longer plain bearings, the already high average unit pressures are greatly increased by pressure concentrations due to deflection. While seizure of the plain bearings has not been common, a number of seizures have occurred, which condition constitutes somewhat of a problem. It is believed that any problems which may arise with the roller-bearing applications can be more readily solved than in the case of plain bearings.

The waterproof grease used in the runner hubs is extremely tenacious, which, together with the slight angular movement of the bearings, should insure retention of a good lubricating film on the rollers. Reverse loads occur on the radial bearings when the wicket gates are closed with the turbine running. However, these loads are kept below the normal loads by the admission of air as described in the paper. The reverse loads do not occur as a blow, but rather change somewhat rapidly as the gates are closed from the speed-no-load to the fully closed position. The time interval is short but it is ample to prevent any action which might be termed a blow.

In general, the author agrees with the comments of Messrs. Nagler and Roberts in regard to venting turbines by air admission at certain gate openings and on shutdowns. Considerable venting of all types of reaction turbines is highly beneficial at low gate openings. Some venting of all fixed-blade turbines at intermediate loads up to nearly the point of best efficiency is also beneficial. From the standpoint of efficiency, venting at and beyond the point of best efficiency is ordinarily questionable, but a moderate amount of air is often used to alleviate the effects of blade and vortex cavitation. Venting becomes of increasing importance at values of the peripheral coefficient ϕ above its best efficiency value, as when operating under reduced heads. With adjustable-blade runners at normal values of ϕ a pressure usually exists under even the lower part of the head-cover cone over practically the entire load range, that is, down to very low gate openings, making it impractical to admit atmospheric air. A pressure lower than atmospheric may sometimes be produced by employing lips upstream of the vent openings. On future turbines, it is proposed to employ a scheme for venting the draft-tube vortex in a manner similar to that shown in Fig. 11 of the paper. The use of a small amount of compressed air has been found beneficial on certain turbines to alleviate blade or blade discharge eddy cavitation.

As Mr. Scoville points out, it is not absolutely necessary to open the blades when starting up an adjustable-blade turbine. However, exceptionally large gate openings, with resulting high thrust loads, are sometimes required to start a unit with the blades flat. The opening feature is certainly a distinct improvement. Cases have come to the attention of the author where the minimum angle of adjustable turbine blades had to be increased on account of starting conditions, particularly on low-head installations. It is also of interest to note in this connection that some of the largest Kaplan turbines in operation have been provided with special extra equipment to give the blades an initial tilt before starting. The 18,000-hp turbines at the Vargon plant in Sweden, with runners 26 ft 3 in. diam, are provided with such equipment,¹⁷ also the 48,000-hp turbines at the Pickwick

¹⁷ "Low Head Produces High Capacity," by George Willock, *Power*, December, 1939, pp. 74-76.

plant¹⁸ in Tennessee with runners 24 ft 4 in. diam. The type of turbine described by the author performs this function automatically without extra equipment, due to the method employed in pivoting the blades.

Mr. Scoville states that Kaplan-type turbines are usually arranged so that the blades close in about 60 sec, and claims this to be an advantage in a plant with a relatively long penstock, the advantage being a lesser pressure rise in the penstock as a result of dropping load. Upon first thought there would appear to be a considerable difference as cited. However, referring to Mr. Scoville's Fig. 16, the time required to drop from full load to one-half load would be reduced in the case of the Kaplan turbine by nearly the same ratio as the reduction in discharge, so that the rate of reduction in discharge per second, which determines the pressure rise, would be only slightly less for the Kaplan than for the automatic adjustable-blade type. In each case it is assumed that the loading and gate timing is the same. In the case of full-load rejection, the total change in discharge is made in the same length of time. Consequently, the average pressure rise would be approximately the same in either case. An exact analysis of the conditions will reveal that, with the automatic type, the rate of reduction in discharge may be slightly higher at the higher gate openings but that the rate reduces somewhat as the gates approach their closed position, resulting in a very desirable action, similar to that produced by cushioning the closing end of the gate stroke.

In every case investigated by the author, the operating characteristics of the automatic adjustable-blade-type turbine, which affect governing, are equal or better than those generally obtained. In this connection, reference is made to the discussion by Mr. Adams in which it is pointed out that the 10,000-hp Austin turbines will be used for the frequency control of a 175,000-hp system, due partly to their excellent governing properties. The WR^2 of each Austin unit is only about 3 per cent of the total of all units of the system. The penstocks at Austin are about as long as any usually encountered with adjustable-blade turbines. From tests, the pressure rise when dropping full load in 3 sec was about 34 ft and the speed rise about 31 per cent.

The necessity for designing the generator for the highest possible runaway speed that could occur, if the blades were deliberately blocked at a relatively flat angle, is somewhat controversial. This question was brought up by Messrs. Scoville and Sharp. At full runaway speed the author's analysis shows that the forces on a blade are principally in the form of a couple, consisting of upward forces near the leading edge and downward forces, of only slightly greater magnitude, near the discharge edge. If that analysis is correct, the couple moment would be the same, irrespective of the location of the axis. That is, there would be a very strong tendency for the blades to open with either the old or new location of the blade axis. However, there are two features of the automatic type which make it inherently less likely for the blades to stick in their highest speed position: (a) The friction moment of the roller bearings used is only a small fraction of that of plain bearings; (b) a low operating fluid pressure is used and the arrangement is such that the fluid cannot become "locked in," as it could in the case of a piston-type valve, such as is employed with the high oil-pressure type of operation.

Referring to Fig. 17, Mr. Sharp seems to have confused the runaway-speed characteristics of the automatic-type turbine with its operation at normal speed under reduced heads. Consequently, his conclusions in regard to the advantages of a Kaplan turbine under subnormal heads are incorrect. High unit speeds, high values of peripheral coefficient ϕ , may be produced

either by overspeed at high heads or by low-head operation at normal speed. It is only in the case of speeds above normal that the blades of the automatic turbine open to reduce the amount of overspeed. When operating at normal speed under reduced heads, the more or less constant pressure applied to the reactive piston causes the blades to assume flatter positions, with respect to gate opening, than their positions at the higher heads. This may, perhaps, be better visualized by stating that, as experimentally determined, the "reactive moment" required for each of the developed designs of the automatic turbine at variable heads and loads is approximately equal to the cube of the runner diameter, times the square of the speed, divided by a constant, $Rb = DN^2/K$. Thus, for variable-head operation at constant speed, the application of a constant reactive pressure tends to make the gate-blade relation vary correctly with the head as well as with the gate opening, in such a way as to maintain both efficiency and power at values approaching those theoretically possible. As a result, the turbine may operate to produce power down to a very low head, the same as is accomplished with the Kaplan type by changing cams, until the value of ϕ equals the highest value obtained with the blades nearly closed. That value may be from 2.3 to 2.5 times its normal value, or even higher, depending upon the number and camber of the blades.

Mr. Sharp also appears to have found from his model tests that a better shape of efficiency curve is obtained as a result of pivoting the blades further downstream than is necessary with the automatic type. The author does not find this to be the case. The results of the Austin tests, Fig. 18 of this closure, rather indicate that there is little increase in the leakage losses through the increased clearances at the periphery of the blades as they approach their open position. The reason for this apparent difference in characteristics may possibly be due either to a difference in camber or to a difference in the shape of the throat ring. The effect of increased clearance may be offset, in the author's opinion, by giving the blades of adjustable runners a little more camber than for fixed-blade runners. With the proportions of throat ring and blade-axis location adopted for the Austin turbines, Fig. 10 of the paper, the maximum clearances of the runner with the throat ring, for the open position of the blades, are kept to quite reasonable values. The throat, minimum diameter of the water passage, is located well below the blade axis, opposite the discharge edge of the blades when open. The clearance at that point is about 2.5 per cent of the runner diameter, i.e., about the same as is used with Kaplan turbines. The corresponding clearance at the leading edge of the blades is about 1 per cent, somewhat smaller than is customary with Kaplan turbines. The smaller clearance at the leading edge should have an advantage in decreasing the danger of a trash jam at that point.

Mr. Sharp has properly called attention to the effect of centrifugal force in giving the blades a tendency to move to their flat-test position. This effect is more pronounced with the wider blades, such as are employed with the four-blade types, but the resulting closing moments about the blade axes are not large when compared with the hydraulic moments. In the case of the automatic-type turbine, it simply has the effect of reducing slightly the required reactive pressure.

Mention was also made of the blade-servomotor capacity. It is interesting to compare the capacities required with the Kaplan and automatic types. The author's analysis of several large Kaplan installations shows that the average required blade-servomotor capacity, based on piston displacement and supply pressure, may be expressed by the formula, $S = 20 PN^{1/4} \div \sqrt{H}$, where S is the servomotor capacity in foot-pounds, P the brake horsepower of the turbine, N , the specific speed, and H the operating head. For the author's type turbine

¹⁸ "A Technical Review of the Pickwick Landing Project," Technical Monograph No. 40, Tennessee Valley Authority, March, 1939, exhibit 21.

a servomotor capacity of $3.33 PN_s^{1/4} \div \sqrt{H}$, is quite ample, or one sixth that of the Kaplan. The Kaplan servomotor is double-acting, while that of the automatic type is single-acting, resulting in a true capacity ratio of 12 to 1, on an energy-supply basis.

The Austin turbines were originally provided with a cam-operated auxiliary-control device to vary the pressure supply to the balance piston, if that was found necessary. The units were at first operated at 20-psi reactive pressure without the use of that device. However, it was found from the preliminary tests that the use of the auxiliary control would result in a substantial improvement in efficiency at certain loadings. The official tests were made with the auxiliary-control device in operation, the actual reactive pressure varying from 14 to 35 psi. Such a device seems to be desirable, particularly for the higher head applications of the automatic type where the greatest variation in the required reactive moment was found from the model tests. The auxiliary control of the reactive pressure is rather simple in design and its use does not in any way affect the several basic advantages of the automatic adjustable-blade-type turbine.

Several important improvements in designs have been made since the paper was prepared. These include an external control valve for varying the reactive pressure, in contrast with the internal valve used at Austin. This valve is operated by an adjustable cam on the gate-operating ring and has a follow-up connection for blade movement. It acts to position the blades positively in accordance with a predetermined gate-blade relation, irrespective of the friction of the runner mechanism and irrespective of the variations in the blade hydraulic moments with load and head that were mentioned in the paper.

Design details have been prepared for a unit rated 40,000 hp 120 rpm 70 ft head to operate under heads varying from 50 to 80 ft. The latter head is at present considered to be the upper limit for the application of the automatic-type turbine.

In closing, the author wishes again to recognize the contributions of the discussers. He also wishes to thank the personnel of the Kanawha Valley Power Company and the Lower Colorado River Authority who cooperated wholeheartedly and to whom much credit is due for the successful applications of this type of turbine.

Production of Seamless Tubes by Combined Effects of Cross-Rolling and Guide Disks

By W. TRINKS,¹ PITTSBURGH, PA.

From the various methods of producing seamless tubes the author has selected the Diescher elongator mill as the basis for discussion in this paper. For a better understanding of this method of tube production, the most recent Diescher mill installation at Allenport, Pa., is described in some detail. Following this the theory of cross-rolling and guide disks in the process of tube manufacture is explained.

AMONG the various seamless-tube-manufacturing processes, the Diescher elongator holds the center of interest because it materializes the long-unrealized dream of the Mannesmann brothers to produce a finished tube by cross-rolling.

In this connection, it will be remembered that, in 1885, the Mannesmann brothers of Remscheid, Germany, introduced their epoch-making invention whereby the production of seamless tubular blanks was made feasible by cross-rolling. This practice is universally known among English-speaking technicians as "cross-roll piercing."

Great hopes were originally expressed for the new rolling process. It was even expected by some that a finished tube could be made in one pass from a round ingot. Not only did this expectation fail of realization, but the use of cross-rolling in the desired heavy reduction of both wall thickness and diameter met the same fate. The cross-roll-piercing process did, however, most spectacularly and effectively deliver comparatively thick-walled blanks, which to be converted into finished tubes required some yet-to-be-discovered procedure. Finally, after many discouraging highly expensive, and time-consuming endeavors, a step-by-step forging process was invented by the Mannesmanns which accomplished the desired tube-forming operations.

This second procedure involves what is universally known as the "pilger mill," which received its name from the fact that the blank passed through the mill with a motion similar to that of the pilger (pilgrims) who, to demonstrate the depth of their piety, went to the shrine at Andernach five steps forward and three steps backward.

EARLY PILGER OR POKE MILLS

In the United States, the early pilger mills were called "poke mills," which term arose from the fact that these early mills were hand-fed and required the operator to push forward or poke the mandrel with the shell on it into the mill after each roll stroke of the bell-shaped pass machined in the rolls.

Those who are familiar with the early history of the production of seamless tubes in the United States will remember that, in the piercing mill, Stiefel, a Swiss engineer employed at the British Mannesmann Works, substituted overlapping truncated conical members, provided with sidewise working faces, for the barrel-type cross-rolls of the Mannesmanns. When Mr. Stiefel emigrated

to the United States, he introduced this highly creditable innovation. During his many years of fruitful life in this country, he was regarded as the dean among seamless-tube experts. Introducing slippage between the rolling faces and the billet permitted the material to flow into length more easily, whereby a reasonably thin-walled blank was produced. To be converted into a product suitable for cold-drawing, this blank required but a few passes in a plug mill. To be converted into an hot-finished product required, in addition to the plug-rolling, a reeling and then a sizing operation. Thus, methods other than cross-rolling had to be used by the Mannesmanns as well as by Stiefel for bringing the pierced blank to final cross section and length.

Almost 50 years elapsed after the Mannesmanns made hollow blanks by cross-roll piercing, before Samuel E. Diescher of Pittsburgh succeeded effectively in elongating a pierced blank into a thin-walled tube by cross-rolling. As in the case of the Mannesmanns, his mill acquired a name descriptive of the operation performed, as interpreted by men skilled in the tube art; this term was "elongator." The elongator has been briefly described in the literature, but the details of the construction, as well as the theory underlying this new method of plastic deformation, have never been published.

In reviewing the rather extensive patent structure which has thus far appeared in public print, the author has discovered that many of the later features of the process are inventions of the originator's brother, August P. Diescher. Therefore, as was the case with the Mannesmanns, here again we have the work of brothers.

DIESCHER ELONGATOR METHOD

The Diescher elongator method requires cross-rolls arranged in a manner generally similar to those of Mannesmann. A pierced blank enters the mill on a freely floating mandrel on which it is

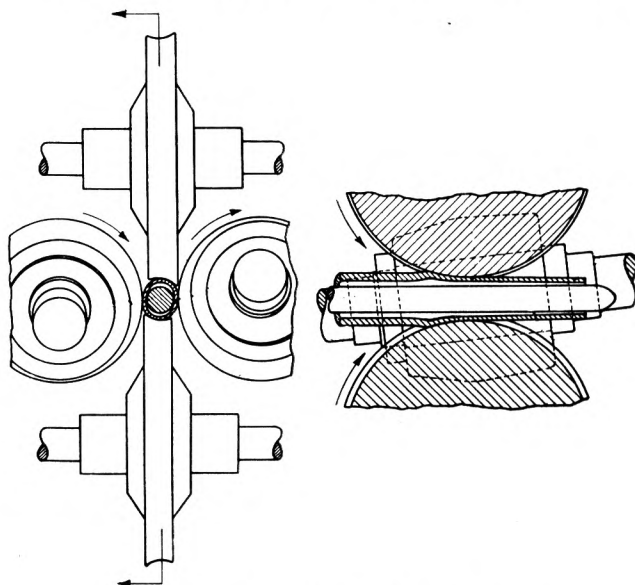


FIG. 1 DIAGRAM OF CROSS-ROLLING PROCESS

¹ Professor of Mechanical Engineering, Carnegie Institute of Technology. Mem. A.S.M.E.

Contributed by the Metals Engineering Division and presented at the Annual Meeting, New York, N. Y., December 2-6, 1940, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

then cross-rolled, Fig. 1. The feature of the mandrel, moving in the forward direction during the procedure, seems to be a departure from the Mannesmann concept. However, that feature alone would not have made the Diescher procedure feasible. To explain this requires first of all a clarification of what occurs in cross-roll deformation.

The cross-rolling process is inherently an expanding process, because the contact area between roll and blank is long in the direction of tube travel, and is short in the direction of roll travel. As a consequence of the laws of plastic flow, the resistance to "bulging" is much smaller than the resistance to elongation of

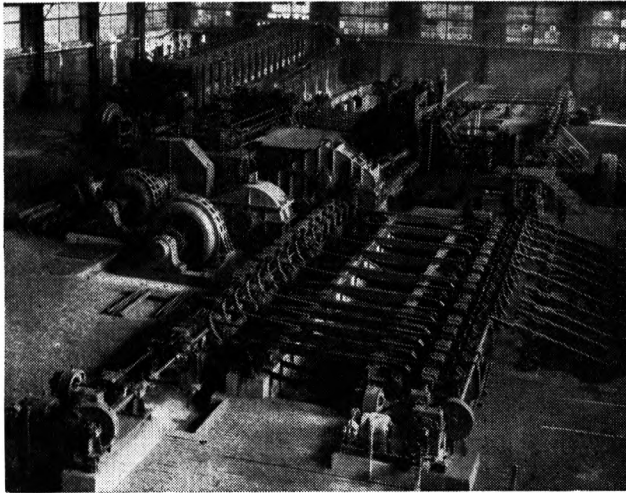


FIG. 2 ELONGATOR UNIT AT PLANT OF PITTSBURGH STEEL COMPANY, ALLENPORT, PA.

the product. As a matter of fact, solely by cross-rolling on a floating mandrel there could be expected at best, under the extent of deformation performed by the elongator, a tube of uncontrolled expansion and of much larger diameter than that of the pierced shell. In the Diescher mill, this expansion is prevented by the elongator disks, which by frictional contact with the blank not only prevent uncontrolled expansion, but also pull the blank forward and convert the expanding tendency of the cross-rolls into elongation of the tube. With any circumferential speed of the cross-rolls and any practicable angularity of the setting thereof, a tube issuing therefrom must travel at a much lower rate of speed than the circumferential speed of the cross-rolls. The inference is natural that the elongator disks should run at a speed but slightly higher than that at which the work material issues, so that the friction will just pull the tube blank along, without doing more work than necessary. However, that inference is incorrect. The circumferential speed of the elongator disks is required to exceed greatly even the circumferential speed of the cross-rolls.

Moreover, there is likely to be drawn another erroneous conclusion. It might be reasoned that, since the elongator disks are intended to prevent tube expansion, they should be set to prevent any and all expansion, so that the finished tube will hug the mandrel tightly. In reality, the work material is allowed to expand away from the mandrel, at the same time, however, requiring precision control of such expansion. Both of these somewhat surprising facts merit discussion. First, however, a detailed description of the equipment will be given to aid in a better understanding of the entire subject.

DETAILS OF LATEST-TYPE ELONGATOR UNIT

Fig. 2 shows the elongator unit in use at the Pittsburgh Steel Company's plant at Allenport, Pa. This installation is the most

recent to be placed in operation in the United States. Also, the Allenport works is the only plant throughout the entire industry at which there may be seen both the Mannesmann and the Diescher practices. The plant equipment includes a large-sized Mannesmann piercing mill, working with comparatively large-diameter ingots, as contrasted with the prerolled rounds used in the Diescher mill. Conjoined therewith is also a pair of pilger mills, which embody the second notable Mannesmann principle of tube making. Further details of the Mannesmann contribution to the art are unnecessary for purposes of this paper since their significance is principally historical.

The photograph from which Fig. 2 is reproduced was taken from a position near the racks where the elongated product is accumulated for crane transportation to locations in the plant where final processings take place as, for instance, cutting to length, sinking, upsetting, cold-drawing, inspecting, and testing. In the upper left-hand part of the illustration appear the heating furnace, the piercing mill, and the transfer equipment from the piercer outlet to the elongator inlet. Of the pair of large motors, appearing nearer the foreground and toward the left-hand region of the picture, the one farther removed from the foreground

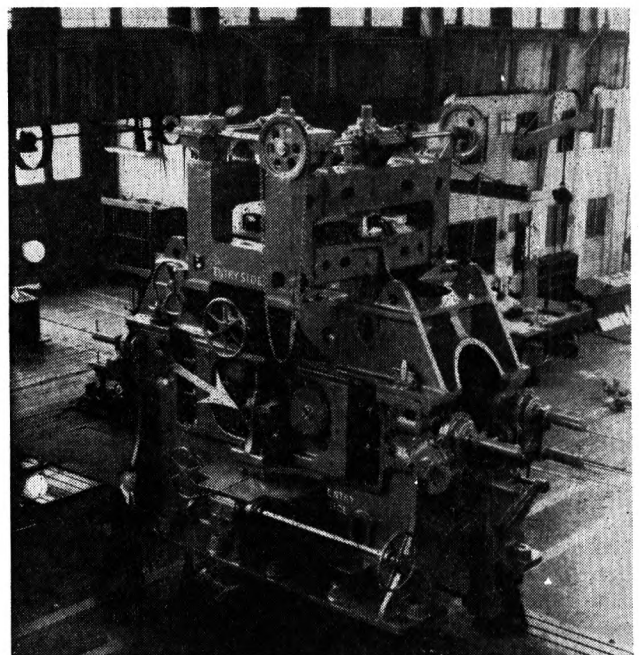


FIG. 3 ASSEMBLY OF ELONGATOR STAND

drives the piercer cross-rolls and the other drives the elongator cross-rolls.

Fig. 3 is a view of the elongator stand while undergoing assembly in the machine shop prior to being set up as part of the elongator unit shown in Fig. 2. In this illustration, the main stand is shown in the reverse position to that which it occupies in Fig. 2, i.e., its entry side is seen in Fig. 3, whereas, its exit side is in view in Fig. 2.

In detail, Fig. 3 shows the main bearings at the nondriven ends of the cross-rolls; the arrangement of main screws, which for the sake of preventing any rocking of the main bearings are applied in pairs; and the screw rig for vertical adjustment of the upper guide disk. Because of the height of this rig above floor level, it is operated by hand chains and so arranged as to encircle a well located at the middle of the housing cap through which crane slings can be lowered for removing the guide disks from their shaft mountings when their renewal is required. Between the

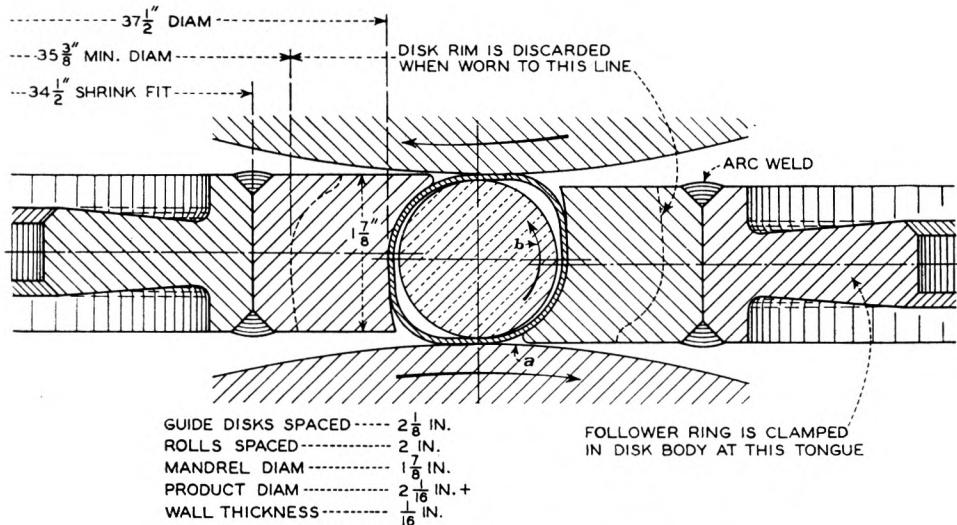


FIG. 4 ACTION OF ROLLS ON THIN-WALLED TUBE

two pairs of main screws, Fig. 3 also shows a window giving access to the guide disks on the side in view, and entry for the guide-disk-drive spindles on the side opposite thereto and not in view.

The hand wheel, appearing centrally below the upper guide-disk-screw rig, is for clamping the upper guide-disk carrier rigidly in position, following its adjustment by the screw rig mentioned. The hand wheel appearing at the left of the one just mentioned is for adjusting the guide disk with its bearings sidewise of the vertical plane of the pass axis, the purpose of which adjustment will be explained later.

Immediately below the hand-wheel adjustment last mentioned and near the bottom of the stand is a companion adjustment which serves in the same manner as the upper one; but in this case in connection with the lower guide disk.

The hand wheel arranged at the right-hand side of the main stand and connected by a horizontal reach shaft to the worm drive, set centrally of the bottom of the housing, is for clamping the lower guide-disk carrier in a manner similar to that previously described as pertaining to the upper guide disk.

Finally, the hand wheel, on the vertical shaft centrally located in relation to the window opening at the right-hand side of the stand, is provided for the vertical adjustment of the lower guide disk. All these adjustments are duplicated on the side of the stand which is hidden from view.

The guide opening, indicated by an arrow, provides entry for the workpiece into the pass located centrally of the stand and formed by the opposed faces of the cross-rolls and of the vertically opposed faces of the guide-disk rims; on the opposite side of the stand there is another such guide which provides for the exit of the workpiece.

Comparing the size of this main stand with the size of the man standing alongside it gives an idea of its massiveness. The stand must be massive to prevent harmful give to the working elements, especially in the working of a thin-walled product accurately to size.

Referring again to the general view Fig. 2, it will be observed from the location of the crossover skids leading from the piercer outlet to the elongator entry, that the workpiece, after leaving the furnace, must travel alongside and beyond the piercer before entering it. After leaving the piercer, then traversing the crossover skids, and entering into and finally passing out of the elongator stand, it will also be noted that, in its elongating course, the workpiece travels in the opposite direction to that in which it proceeded through the piercer. This procedure was adopted

because with thin-walled piercing, the shell tends to become somewhat smaller in diameter at its exit end than throughout its general course, which created a condition inviting chilling under the closer approach of the walls at this narrowed end to the elongator-mandrel surface. By causing that end to enter the guide-disk pass first, there is not the amount of time for chilling to take place that would occur if the direction of workpiece travel for both piercing and elongating was the same.

Obviously, before the pierced shell may be entered into the elongator, a mandrel bar must be strung through it. In the right-hand background of Fig. 2 may be seen a number of these bars resting on the mandrel-cooling or storage bed. As soon as a mandrel has been advanced from the bed to its position within the pierced shell, both shell and mandrel enter the elongator pass and are fed through it, traveling along between the elongator-drive spindles, thence through a passageway provided in the drive-gear housing, which can be seen in the center of the illustration, and finally forward along the runout table the drive of which appears in the left-hand foreground. By flag-switch control, the three-arm star-throwout rig, appearing immediately next this runout table, directs the finished tube with the mandrel still inside it across skids to a chain transfer and from that mechanism to another table which is parallel with the runout table but operates in the reverse direction. This latter table passes the tube and mandrel toward the rear to a pinch-roll stand which can be seen at the end of the table. At this point, the mandrel is extracted from the tube and proceeds along its course and is finally, by means of a star throwout, transferred to the mandrel storage and cooling rack.

By means of another star throwout, the tube from which the mandrel was extracted is moved sidewise into the finished-product storage rack, shown at the right, from which the product is removed in crane-load batches for further processing, as previously described.

Beyond this tubular product storage rack and midway between it and the background is shown the elongator guide-disk-drive motor.

THEORY OF CROSS-ROLLING SEAMLESS TUBES

In a vertical plane, the cross-rolls are set at an angle of 6 deg with the direction of travel of the tube. Ordinarily and with no allowance for slip between cross-rolls and tube, the latter would travel ($\sin 6 \text{ deg}$) times the circumferential roll speed. A typical roll speed is 800 fpm, which makes the tube-delivery speed equal

POWER REQUIRED FOR ROLLING OPERATIONS

With regard to power consumption, the following data are available. The rolling of thin-walled tubes requires more power per unit of volume times elongation in unit time than the rolling of the thick-walled product.

The power required per net ton of product also varies with the size of product being made. For example, in rolling common-steel pierced blanks of 3 in. diam and 0.266 in. wall to a product of $2\frac{5}{8}$ in. diam and 0.125 in. wall at the rate of about 1.7 fps out of the elongator pass and produced at the rate of 240 pieces 20 ft long per hr, there would be required about 800 kw of power for all drives, including table drives of the elongator.

Among data regarding the consumption of power in elongating per se, and therefore dealing solely with that required for driving the cross-rolls and guide disks, the following figures should be of value: In elongating to 0.09 in. wall thickness, about 95 hp are required per cu in. of metal displaced per sec; whereas, in elongating to double that thickness about 70 hp are required per cu in. displaced per sec, the guide disks consuming about 25 per cent of the power consumed by the cross-rolls.

Closely allied with disk wear and power consumption is the cost of tooling. The smaller the diameter and the wall thickness of the product, the higher is the cost of tooling per ton of product. At present, the cost of all tooling in rolling the $2\frac{5}{8} \times \frac{1}{8}$ -in. wall tubing, mentioned under power consumption, is about \$1.10 per net ton of product. This includes all items such as rolls, disks, mandrels, etc., and is expected ultimately to be substantially reduced.

For the sake of completeness, additional questions must be

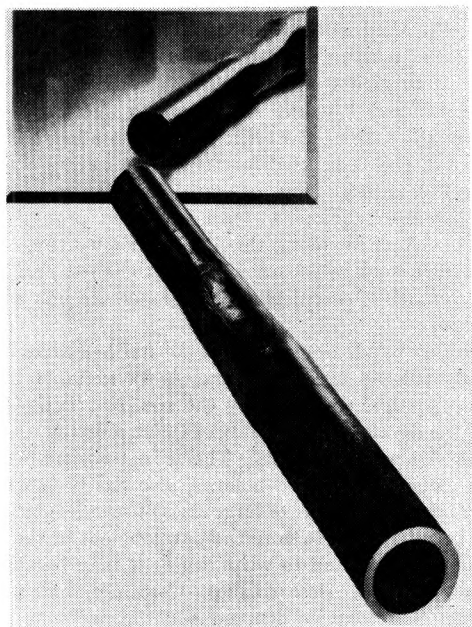


FIG. 6 TUBE PARTIALLY ROLLED FROM ECCENTRIC BLANK

discussed briefly; one is the diameter of the cross-rolls. They must be rigid enough to roll the thinnest practical tube without noticeable deflection, even if the temperatures among the blanks vary. Designers prefer to be on the safe side and make them somewhat larger than the needed neck diameter implied, rather than too small. The next question is that of greatest possible elongation. The Diescher elongator has rolled with 5 elongations, which means that the delivered tube is 5 times as long as the entering hollow blank. In practice, such extreme elongations

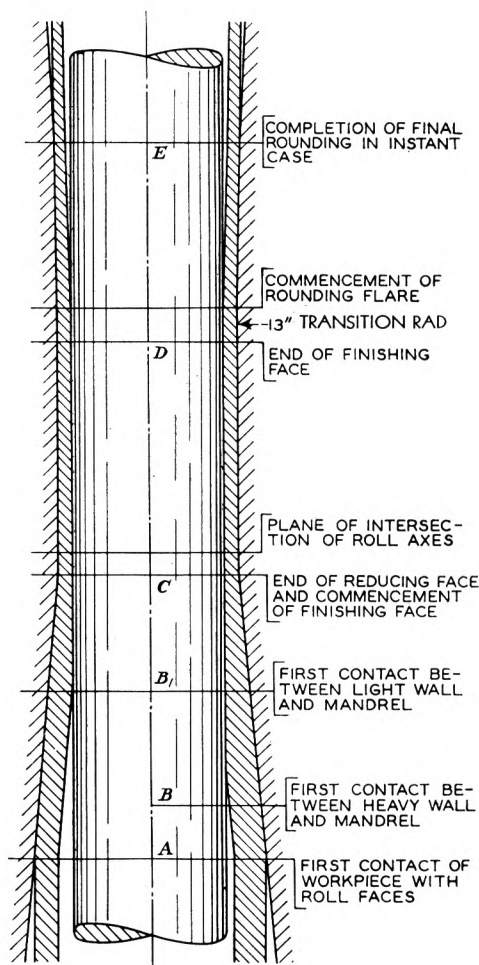


FIG. 7 DIAGRAM OF STAGES IN ROLLING OPERATION

are not used. The average elongation is about 2, which means that the tube has twice the length of the hollow blank. In this manner, piercer output and elongator output are about alike, and the highest rate of combined production results.

CONCENTRICITY OF DIESCHER MILL TUBES

The elongator mill has gained a reputation for producing concentric tubes. This statement may be discussed from two angles, viz., the importance of concentricity, and the reasons why the Diescher mill attains it. The importance of concentricity is realized by all consumers of tubing and therefore need not be stressed here. The next question is: Does the elongator produce concentric tubes and, if so, why? The fact that it produces concentric tubes may be judged from Fig. 6, which shows a tube that had been rolled part way from an eccentric blank. The latter had been bored out of center. The reason for rolling a bored blank instead of a blank produced by cross-rolling is that a uniformly eccentric blank cannot be obtained from a piercing mill because, in the latter, the piercing plug meanders, following fortuitous soft spots caused by nonuniform heating or segregation.

In turning now to reasons for the concentricity of tubes rolled on the elongator mill, it is conceivable that, when the tube blank first enters, an eccentric blank would vibrate the mandrel with high frequency, whereby resisting forces are produced which, in conjunction with the well-known fact that thick walls are more easily deformed than thin walls, tend to reduce the eccentricity of the workpiece. Furthermore, a thick wall portion introduced

into the cleft between the rolls and the mandrel emanates therefrom more pronouncedly than its companion thinner wall portion, rubs harder against the elongator disks, becomes hotter, and is more easily turned into length. By the combined effect of these actions, the mandrel becomes centered by the time the tube is traversing the long finishing pass, which in Fig. 7 reaches from C to D . From then on, the remainder of the tube (which means almost the entire length) is forced through the spaces between the rolls and a concentrically held mandrel, whereby concentricity is obtained. It goes without saying that concentricity as used in practice is a relative term, and that maximum approach to perfect concentricity requires maximum rigidity of the mill.

CONCLUSION

In conclusion, we may consider the economic side of the matter: Under what conditions should existing equipment be replaced by an elongator mill? Up to the present time, this question has not arisen in practice, because the four mills which are now in operation were installed as additional equipment for the purpose of making a product which heretofore could not be made, i.e., a hot-rolled, seamless, concentric tube with very smooth surfaces. Upon inquiry the author discovered that the patent owner, the Diescher Tube Mills, Inc., has a very commendable policy; new licenses seem unlikely to be solicited unless the existing mills cannot cope with the demand.

Discussion

A. B. Cox.² In the Diescher tube-rolling machine, as in steel-rolling processes in general, successful operation is dependent upon relative values of external and internal friction. The great importance of this principle in the field of rolling materials may therefore justify some comment on this phase of the paper.

At one point the author states that the theoretical derivation of relative speeds of cross rolls and longitudinal rolls is based on the assumption that the coefficient of external friction is independent of the speed, i.e., relative motion or slip, which previously had been taken as zero. The writer knows of no experimental data which would justify this assumption of a constant coefficient of friction for all speeds. Regardless of whether the friction is dry or lubricated, experimental data show that the coefficient of friction varies very widely with speed.³ If the coefficient of friction between hot and cold steel is constant over any considerable range of speed, a citation to the data which show this should be of interest.

The author also refers to the laws of plastic flow, which brings up the subject of internal friction. It is well known that, in general, the resistance of a solid to deformation varies with the rate of deformation. The ease of flow, or "flowability" of the material, increases with increase in the rate of deformation; at least for low and intermediate rates of deformation. Conversely, the coefficient of viscosity (or coefficient of internal friction) decreases.⁴ This is true of all materials for which data are available. The coefficient of viscosity for all materials, solid, liquid, or gaseous, follows the same law. It is only when data are taken over a relatively limited range of speed that the coefficient appears to be approximately constant in some cases. This is true even for water and for air. Hence, it is extremely probable that the coefficient of internal friction of steel, no matter how cold or how hot, varies greatly with rate of deformation.

² South Hills, Pittsburgh, Pa.

³ "Journals and Bearings" section, L. S. Marks, *Mechanical Engineers' Handbook*, third edition, 1930. Stribeck data for a lubricated journal, pp. 243, 244.

⁴ Refer to Bibliography on subject of "Friction," by Committee E-1, American Society for Testing Materials, 260 S. Broad Street, Philadelphia, Pa.

Given experimental data for the curve of variation of speed versus coefficient of internal friction of steel at the temperature at which the metal is to be worked, and the curve of variation of the coefficient of external friction versus speed for hot or cold steel, would it not be possible to predetermine the relative speeds of the cross rolls and the elongator rolls of the Diescher machine for optimum performance with engineering certainty? It is the common impression among steel men that all mills of this general type are hard to get started in successful production on any but a product which has already been fully standardized in production. Apparently this is due to the difficulty of obtaining just the right relative tangential and longitudinal speeds. If the author can supply information which will refute this general opinion, it would be of value both to the tube manufacturer and to the manufacturer of the tube-rolling machine.

C. W. LITTLER.⁵ From the writer's investigations in the matter, it is evident that the Diescher elongator has contributed greatly to the advancement in the art of seamless-tube manufacture. This is particularly true with reference to the ability of such a mill to maintain uniform wall thickness within very close tolerances. It is understood that elongations of 4 to 1 have been accomplished. No doubt the rotary disks add greatly to this possibility. The uniformity achieved on such a tube coming from the elongator has a beneficial effect on the reducing mill.

G. A. PUGH.⁶ The Diescher elongator finishes pierced blanks into seamless tubes, eliminating intermediate operations such as pilgering, plug rolling, and reeling. The plug-rolling and reeling intermediate operations are employed by most of the tube manufacturers in this country.

The urge to eliminate the plug-rolling mill, as a means of elongating pierced hollows, is a natural one, since the plug-rolling operation is irrational and contributes most of the difficulties usually considered inherent with the manufacture of seamless tubes. For the sake of clarity, it should be understood that the Diescher mill has been used for tube sizes having a maximum diameter of about 4 in. The hot finishing of tubes of light wall, smaller than 4 in. diam, has been a difficult problem and the Diescher development along this line is noteworthy. Qualities of the product, such as accuracy of wall thickness and reduction in the percentage of eccentricity, have made the unit a commercial success.

On the other hand, large-diameter hot-finished tubes have been successfully finished by cross rolling, as for example, by the rotary rolling method employed by the National Tube Company. As one of the developments of R. C. Stiefel, mention was made of it in a paper⁷ presented in 1928. The use of a second piercer, as a means of elongating pierced hollows, also has been widely employed by all of the makers of large sizes of seamless tubes.

It will be interesting to note the development in the Diescher mill as time goes on and to what degree it may be adapted for the production of large sizes of tubes. Certainly, there must come a day when the economic demand and the necessity for closer wall tolerances will force engineers to adopt methods which make such attainments possible.

C. R. SADLER.⁸ This paper has been of particular interest to the writer who was closely concerned in the installation of the

⁵ Chief Engineer, Jones & Laughlin Steel Corporation, Pittsburgh, Pa. Mem. A.S.M.E.

⁶ Youngstown, Ohio. Mem. A.S.M.E.

⁷ "The Manufacture of Seamless Tubes," by R. C. Stiefel and G. A. Pugh, *Trans. A.S.M.E.*, vol. 49-50, 1927-1928, paper IS-50-7, pp. 17-22.

⁸ American Munitions Division of American Type Founders, Inc., Elizabeth, N. J.

first elongator, and for several years was personally familiar with its operation. As the author states, this is the first time the theory underlying the "elongator" method of producing tubes has been published. The statements in his paper are borne out by the writer's experience.

While this paper makes mention of the concentricity of the tubes produced by this process and refers to tests made with an eccentrically drilled blank, the actual wall measurements of the blank in question were not stated. It may be of interest to know that these wall dimensions were as follows:

Before processing.....	{ 0.120 in. minimum 0.358 in. maximum
After processing.....	{ 0.095 in. minimum 0.104 in. maximum

Another advantageous feature of the elongator method is the smooth interior surface of the product. Internal scratches have always been the bugbear of the seamless-tube industry, and no satisfactory method has been devised for their removal. For this reason, a method of hot rolling was sought in which scratches could not occur. Naturally, attention was turned to the idea of rolling tubes over a smooth bar rather than over a plug. Of the various methods of accomplishing this, the elongator method has proved to be the most satisfactory. The reason for this will be apparent from a study of Fig. 1 of the paper. It will

be noted that there is but a slight sliding movement between the tube and the bar, and that the tube is in momentary rolling contact with the bar at only two points during the operation. Thus, there exists no tendency to produce scratches or scores in the tubes. Not only is this a very desirable feature in the production of hot-finished tubes, but where tubes are subsequently cold drawn it results in the production of superior tubes with fewer cold-draw passes.

Another feature is that, while the process is suitable for any size tube, smaller hot-finished tubes have been successfully produced by this method than were possible before its introduction. The advantage of this will be apparent to any tube manufacturer who has had experience with the difficulties inherent in hot sinking.

AUTHOR'S CLOSURE

The constructive character of the discussion which has been offered on the present paper is very much appreciated. Additional facts have been presented and they are helpful in the understanding of the process.

Mr. Cox recommends that additional research work be done on the friction between blank and rolls and also on the internal friction within the blank. Information on these problems has been accumulating for some time but as yet is not adequate for presentation.

The Flow of Saturated Water Through Throttling Orifices

By M. W. BENJAMIN¹ AND J. G. MILLER,² DETROIT, MICH.

In this paper are presented the results of tests to determine the flow characteristics for saturated water and for various mixtures of saturated water and steam through sharp-edged thin-plate orifices. Tests show that the actual flow of saturated water through these orifices is considerably greater than would be expected from theoretical calculation based upon a change of state and that no critical back-pressure condition is evident over the range of initial pressures considered. Primarily, this investigation was intended to determine the feasibility of using throttling orifices alone or in combination with float-operated drainers for regulating the draining of condensate from feedwater heaters. The tests which form the basis of the paper have provided sufficient information to permit the derivation of practical design formulas, which have been used successfully in several instances by the authors' company. These test data apply only to throttling orifices and should not be used to design orifices for metering purposes. An Appendix to the paper shows the application of the formulas to the design of a single-stage orifice to drain the condensate from a feedwater heater, and to the design of an orifice to be used in series with a float-operated drainer.

EARLY in 1939, the authors became actively interested in the flow of boiling water through pipes in connection with the rapid erosion of elbows in heater drain piping on a 60,000-kw steam turbine. After some study, it was found that erosion was a function of the amount of flashing and the quantity of water flowing in the pipe, and interest in the subject was intensified by the indication that certain operating difficulties with float-operated condensate drainers also might be traced to the phenomena encountered with water flashing into steam in the drainer discharge pipe.

In a paper³ by Bottomley, the suggestion was made that orifices could be used in place of float-operated traps for draining feedwater heaters. Such an application would eliminate the operating troubles with traps and, if the orifices were installed at the end of the drain line rather than at the beginning, would prevent erosion in the piping.

While the theoretical analysis of the flow of saturated water (water at saturation temperature and pressure) through orifices, which assumes a change of state in the orifice, seemed straightforward enough, the limited published test data showed the actual capacity of an orifice passing saturated water to be several times greater than its theoretical capacity. Since the available test

data were not complete and because practical coefficients were needed for actual design purposes, it was decided to conduct an experimental investigation of the flow of saturated water through sharp-edged thin-plate orifices to determine whether it would be practicable to use such orifices in lieu of traps for draining feedwater heaters.

While the tests which form the basis of this paper do not cover all the variables which might have been investigated, they are sufficiently complete to provide a practical basis for design that has been applied successfully in several instances.

In this paper the term "saturated water" is used in preference to "hot" or "boiling" water as used by earlier writers, since it is considered to be more definite and possibly more accurate. It is used to denote water at saturation pressure and temperature, and refers to the condition of the water on the upstream side of the orifice.

COMPARISON OF THEORETICAL AND ACTUAL FLOW OF SATURATED WATER THROUGH ORIFICES

The theory of the flow of saturated water has been thoroughly covered in several published papers⁴ and, therefore, will not be repeated here.

For purposes of comparison, Fig. 1 shows the theoretical and actual flow of saturated water through orifices for an initial pressure of 145 psi abs and back pressures ranging from 0 to 145 psi abs, while Fig. 2 shows the maximum theoretical and actual flow through orifices for initial pressures ranging from 14.7 to 300 psi abs and a constant back pressure of 14.7 psi abs.

EXPERIMENTAL INVESTIGATION OF FLOW OF SATURATED WATER THROUGH ORIFICES

Fulfillment of the purpose of the test required that several points be kept in mind concerning the design of the orifices and test equipment, as follows:

- 1 That the draining of condensate from a higher-pressure to a lower-pressure feedwater heater is primarily a throttling process.

- 2 That the controlling factors to be considered in designing an orifice for draining condensate from one heater to another for maximum load on a turbine are (a) the pressure differential between the heaters; (b) the initial temperature and pressure of the condensate; and (c) the quantity of condensate to be drained from the higher-pressure heater. The initial temperature of the condensate will be the saturation temperature corresponding to the initial pressure in all cases except those in which there is undercooling. The effect of undercooling on the flow of the condensate through an orifice is in general the same as the effect of a static head on the upstream side.

⁴ "Experimental Researches on the Flow of Steam Through Nozzles and Orifices," by A. Rateau, D. Van Nostrand & Co., New York, N. Y., 1905, supplementary chapter at end of book, pp. 62-74.

"Discharge Capacity of Traps," by A. E. Kittredge and E. S. Dougherty, *Combustion*, vol. 6, September, 1934, pp. 14-19.

"Fluid Flow Through Two Orifices in Series," by Milton C. Stuart and D. Robert Yarnall, *Mechanical Engineering*, vol. 58, 1936, pp. 479-484.

"Flow of Boiling Water Through Orifices and Pipes," by W. T. Bottomley, *Trans. North-East Coast Institution of Engineers and Shipbuilders (England)*, vol. 53, 1936-1937, pp. 65-100.

"The Flow of Hot Water Through a Nozzle," by B. Hodgkinson, *Engineering (London)*, vol. 143, 1937, pp. 629-630.

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³ "Flow of Boiling Water Through Orifices and Pipes," by W. T. Bottomley, *Trans. North-East Coast Institution of Engineers and Shipbuilders (England)*, vol. 53, 1936-1937, pp. 65-100.

Contributed by the Power Division, and presented at the Annual Meeting, New York, N. Y., December 2-6, 1940, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

3 That all of the factors given under (2) decrease with a drop in load on the turbine; and it is possible that an orifice designed for full load will be larger than needed to pass the condensate at reduced load even though the pressure differential across the orifice also is reduced. In such a case, some steam will be cascaded to the lower-pressure heater along with the condensate. This means that more steam will be bled from the higher-pressure tur-

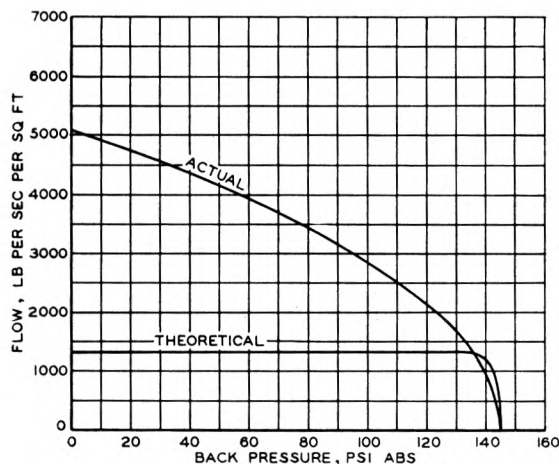


FIG. 1 THEORETICAL AND ACTUAL CHARACTERISTICS FOR FLOW OF SATURATED WATER THROUGH ORIFICES
(Initial pressure, 145 psi abs.)

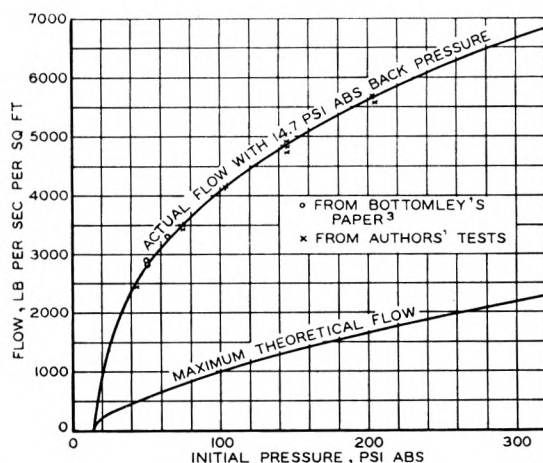


FIG. 2 ACTUAL FLOW AND MAXIMUM THEORETICAL FLOW OF SATURATED WATER THROUGH SHARP-EDGED ORIFICES
(Initial pressures ranging from 14.7 to 300 psi abs and a constant back pressure of 14.7 psi abs.)

bine stage than is necessary for its own stage of feedwater heating, which in turn may reduce the turbine cycle efficiency somewhat, especially if the amount of steam cascaded is excessive.

Therefore, in order to provide the necessary test data for designing orifices to drain the condensate from feedwater heaters, the test equipment was built to permit the determination of (a) the flow of saturated water through orifices for various initial and back pressures; (b) the effect on the flow of saturated water through orifices produced by a static head above the orifice (undercooling); and (c) the effect on the flow of passing steam through the orifice along with the water.

Design of Orifices. While it is not an established fact, it is expected that long-continued throttling may produce wear or erosion of the orifice with consequent passage of steam at all turbine loads. To prevent loss of cycle efficiency it might be neces-

sary to make periodic replacements and, for this reason, any heater drain-line orifice should be simple to make and easy to replace in service. If the orifice plate is made of corrosion-resisting steel, experience shows that, where no change of state occurs, there will be little if any wear of the sharp edge of the orifice; however, with orifices installed in hot-drip systems, considerable erosion due to flashing or cavitation has been noted on the downstream face of the orifice plate. It should be noted here that drain-line orifices are not intended for metering purposes.

Fig. 3 shows the design and Table 1 gives the diameters of the orifices used in this investigation. These orifices were installed in a horizontal 6-in. pipe.

TABLE 1 DIAMETERS OF ORIFICES USED IN INVESTIGATION

Orifice number	Orifice diameter, in.
1.....	0.247
2.....	0.369
3.....	0.364
4.....	0.503
5.....	0.614
6.....	0.707
7.....	0.879

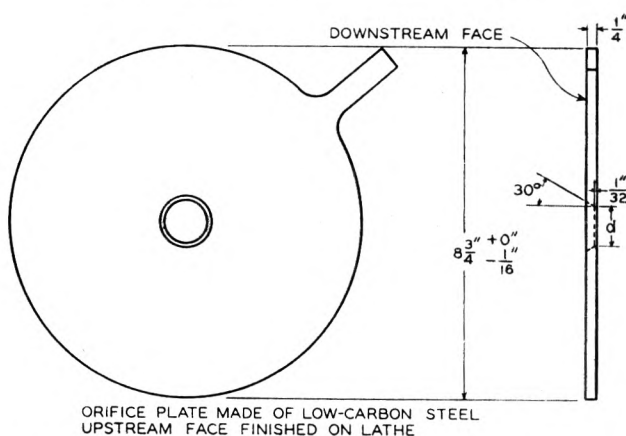


FIG. 3 DESIGN OF ORIFICES USED IN TESTS OF FLOW OF SATURATED WATER

Description of Test Equipment. Equipment used in the orifice tests consisted of the following:

- 1 A flash tank for regulating the pressure and supply of saturated water.
- 2 A set of orifices and a flanged holder for them.
- 3 A heat exchanger.
- 4 Two weigh tanks—one for the condensate and another for the heat-exchanger cooling water.
- 5 Temperature-measuring apparatus.
- 6 Pressure gages.

The schematic arrangement of the equipment and the relative location of thermocouples and of pressure gages is illustrated in Fig. 4.

Fig. 5 shows an orifice clamped in the flanged holder. The short glass filler immediately downstream from the orifice was used to permit visual observation and photography on some of the runs. During most of the tests, however, a steel filler was used.

All temperatures were measured with iron-constantan thermocouples, calibrated for 8 in. immersion and a reference junction temperature of 32 F. Calibration of the couples is believed to be accurate within ± 0.5 F and, because of the depth of immersion and the precautions taken to prevent air circulation around the couples in their wells, it is believed that the measured temperatures are in error by no more than ± 0.5 F. The design of the thermocouple well is shown in Fig. 6.

Operation of Equipment. The controlling conditions in operat-

FIG. 4 SCHEMATIC LAYOUT OF EQUIPMENT FOR TESTING THE FLOW OF SATURATED WATER THROUGH SHARP-EDGED ORIFICES

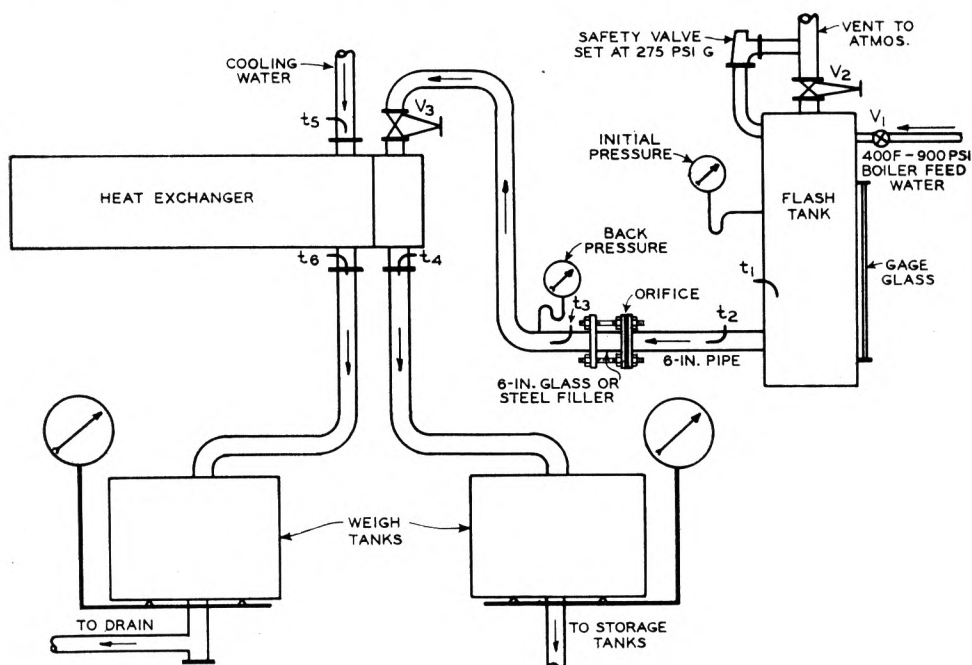
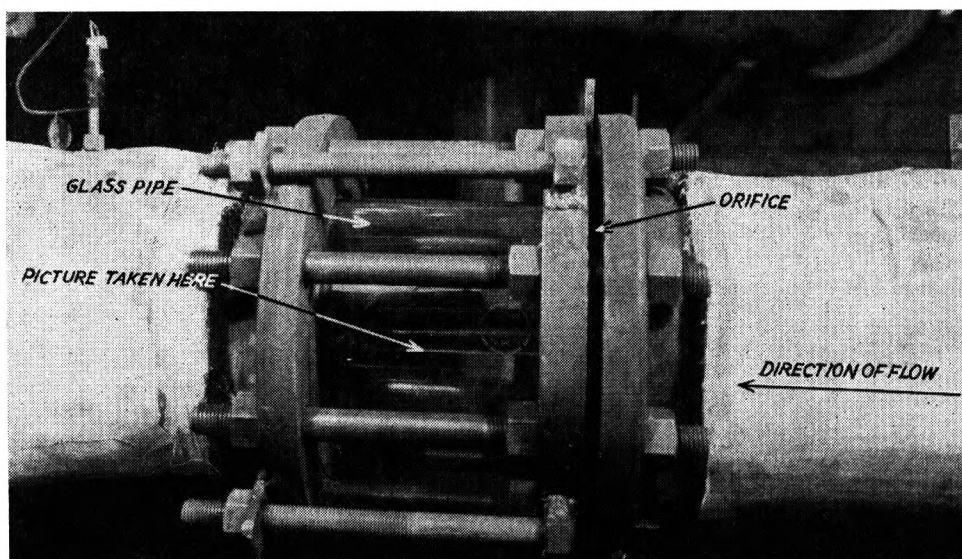


FIG. 5 ORIFICE AND GLASS-FILLER ASSEMBLY



ing the test equipment were the pressure and water level in the flash tank and the back pressure on the orifice. For any given test, the required combination of these conditions was obtained by balancing the flow of 400-F water into the flash tank through valve V_1 against the flow of flashed steam through vent valve V_2 and of water or a mixture of water and steam through the orifice and back-pressure control valve V_3 . Usually about 15 min were required to obtain the desired combination of pressures and water level, but once this condition was established slight adjustment of the valves was sufficient to maintain it because of the relatively constant pressure and temperature of the water supply. For the tests in which the orifice was passing both water and steam, the water level remained constant at about the center line of the horizontal 6-in. pipe and was of secondary importance. However, for the tests in which only saturated water passed through the orifice, the water level was established at from 3 to 8 in. above the center line of the orifice and was maintained at the established level within $\pm 1/2$ in. In the tests to determine the flow with a

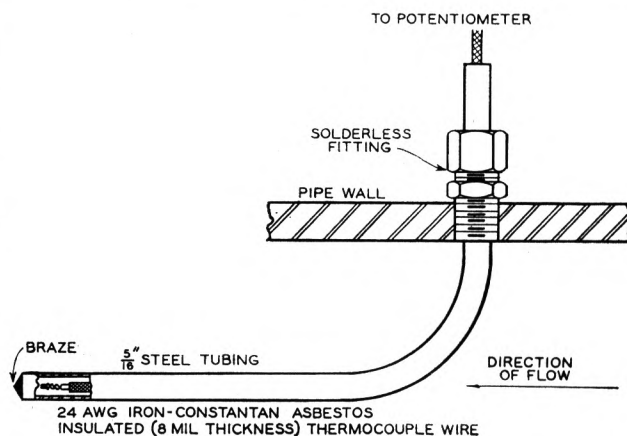


FIG. 6 DESIGN OF THERMOCOUPLE WELL USED IN TESTS TO DETERMINE FLOW OF SATURATED WATER THROUGH SHARP-EDGED ORIFICES

static head above the orifice, the flash tank was filled with water under a pressure greater than the saturation pressure corresponding to the water temperature. However, in all cases, the inlet-water temperature was above the saturation temperature corresponding to the back pressure on the orifice.

TEST RESULTS

The test data give the actual flow of saturated water through sharp-edged thin-plate orifices for various initial pressures and show the effect on this flow of (a) varying the back pressure on the

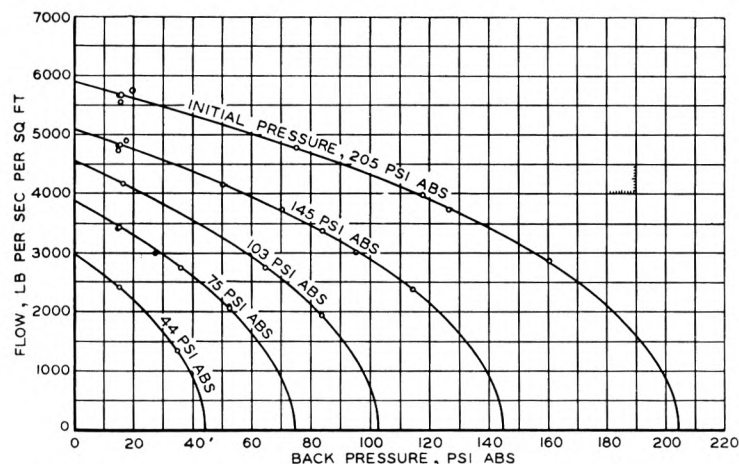


FIG. 7 ACTUAL FLOW OF SATURATED WATER THROUGH SHARP-EDGED ORIFICES

orifice, (b) passing steam along with the water, and (c) a static head above or undercooling before the orifice.

The actual flow of saturated water through orifices, on the basis of pounds per second per square foot of orifice area as found on test for five initial pressures and various back pressures, is presented in Fig. 7. The results given were obtained from tests of

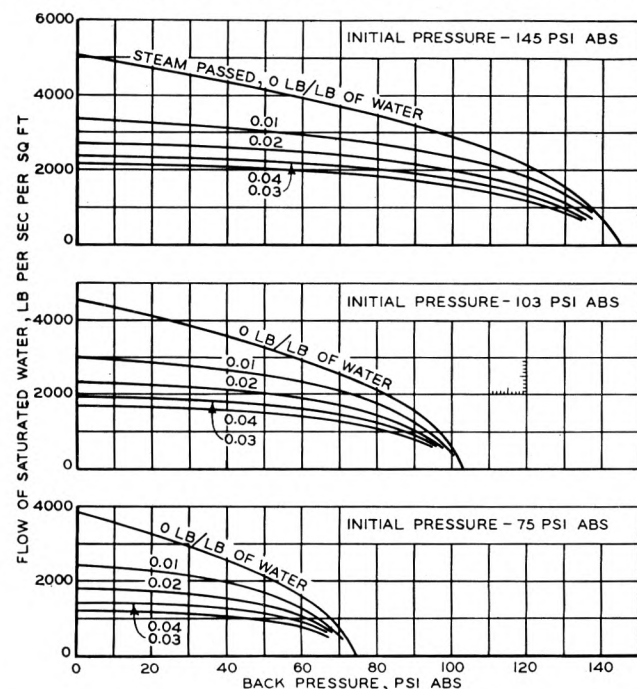


FIG. 9 ACTUAL FLOW OF SATURATED WATER THROUGH SHARP-EDGED ORIFICES WHEN PASSING VARIOUS MIXTURES OF STEAM AND WATER

all the various sizes of orifices listed under "Design of Orifices."

Fig. 8 shows the flow of saturated water when a mixture of steam and water is passed through an orifice. The curves are arranged to show how the initial pressure, back pressure, and relative amounts of steam, included in the mixture, affect the

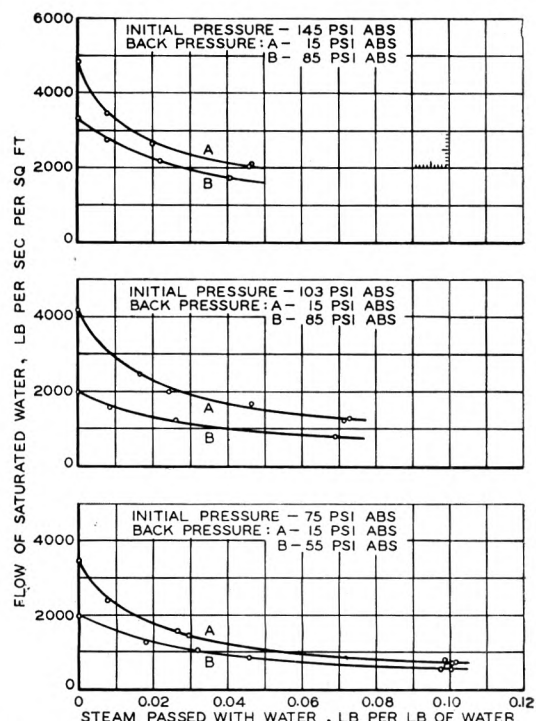


FIG. 8 ACTUAL FLOW OF SATURATED WATER THROUGH SHARP-EDGED ORIFICES WHEN PASSING MIXTURE OF STEAM AND WATER

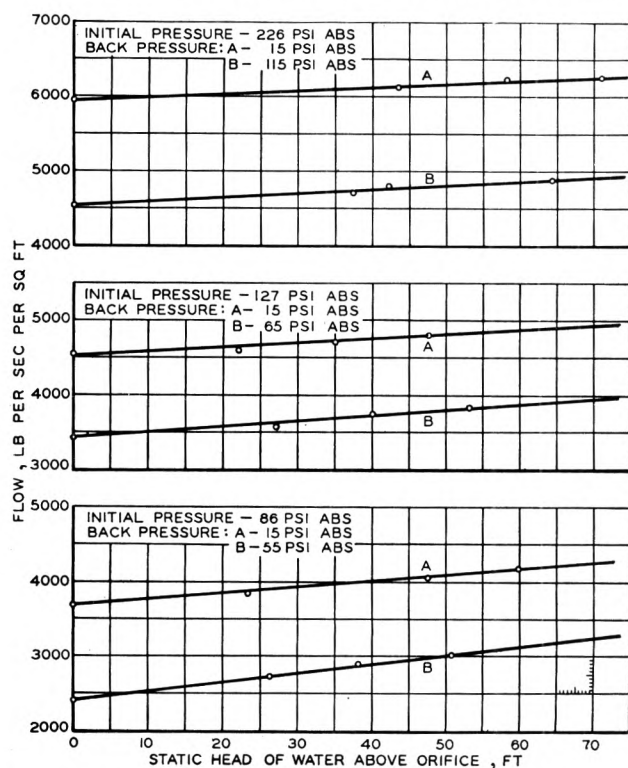


FIG. 10 EFFECT OF STATIC HEAD ON FLOW OF SATURATED WATER THROUGH SHARP-EDGED ORIFICES

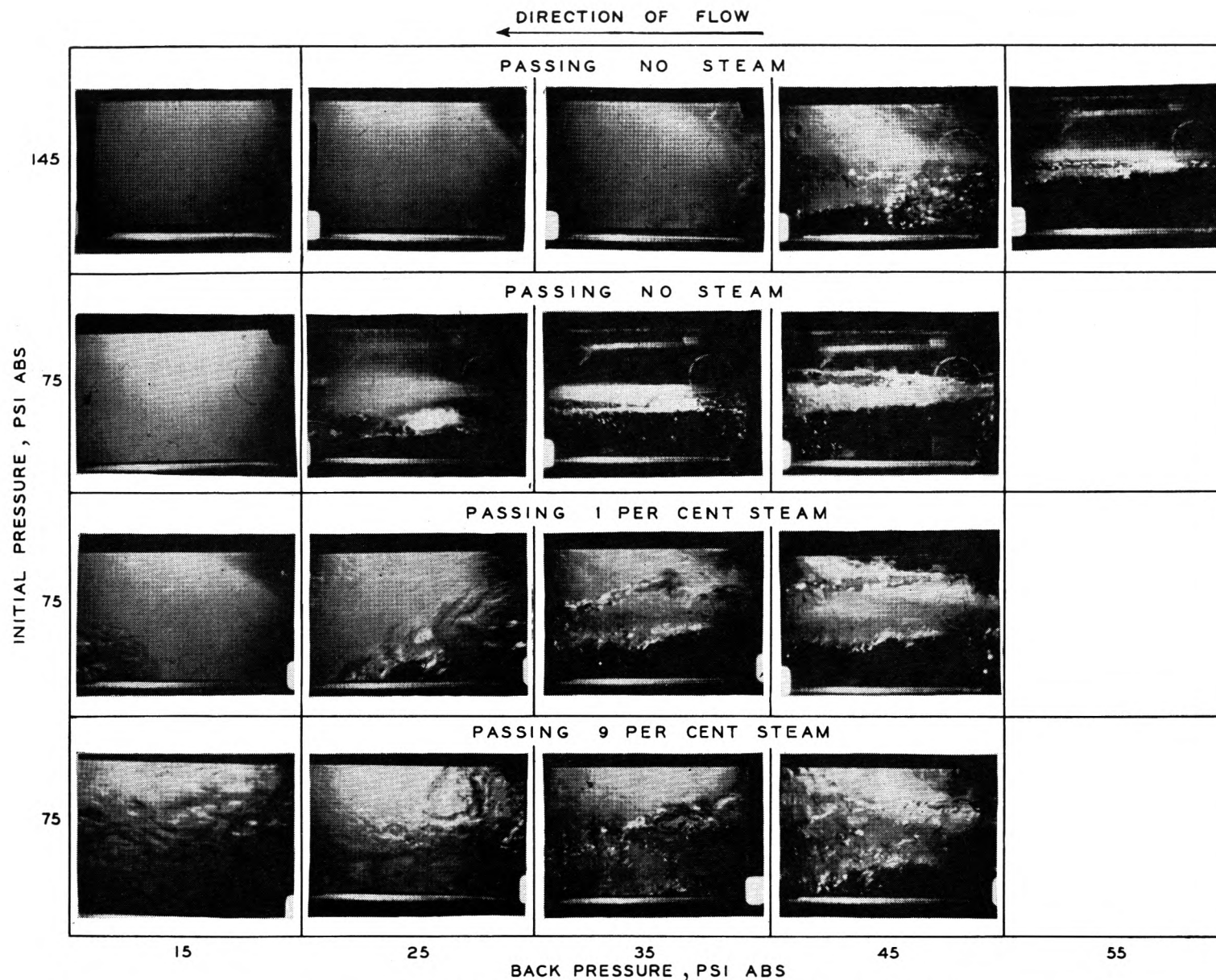


FIG. 11 MIXTURE OF STEAM AND WATER LEAVING DOWNSTREAM FACE OF ORIFICE
 (Each picture represents one test, the initial pressure for which is given in the ordinate and the back pressure in the abscissa. The direction of the flow is from right to left. The screen shown in the pictures is a guard around the glass filler.)

flow. It is important to note that the amount of steam passed is given as a fraction of 1 lb per lb of water and that the total weight actually passed by the orifice is the sum of the weights of water and steam. Fig. 9, obtained by combining the data for initial pressures of 75, 103, and 145 psi abs from Fig. 7 with the data from Fig. 8, shows clearly and in convenient form the effect of passing steam with the water.

Should the rate of supply exceed the capacity of an orifice, a static head of water will build up above the orifice. Therefore, the water at the orifice center line will be under a pressure in excess of the saturation pressure corresponding to the temperature by the amount of the static head. Under this condition the fraction of water flashed into steam in passing to a region of lower pressure will be exactly the same as though the orifice were merely submerged, but the capacity of the orifice is increased somewhat as a result of the increase in pressure drop across it represented by the static head. An equivalent case is one in which the orifice is passing hot water at a normal level but undercooled to some extent below the saturation temperature. For instance, cooling the water from a saturation temperature of 332.1 F down to 317.1 F, while maintaining the original saturation pressure of 106 psi abs, is the same condition as having an equivalent static head of 20 psi (50.7 ft) above saturation pressure with water at 317.1 F. Consequently, the test data for both conditions are presented on the basis of a static head above the orifice. Fig. 10 shows how static heads of from 0 to 70 ft affect the flow at three different initial saturation pressures.

The photographs, reproduced in Fig. 11, show the flow leaving the downstream face of the orifice for two initial pressures and various back pressures. The two upper sets of pictures are from tests in which no steam was passed with the water, while the two lower sets are from tests in which steam was passed through the orifice with the water as indicated. In the two upper groups of pictures, it is interesting to note that, for the higher back pressures, the flow leaves the orifice in the form of a jet with practically no flashing evident within the length of the glass filler. As the back pressure decreases, however, flashing occurs nearer the orifice as shown by the breaking up of the jet, and the lower the back pressure the nearer to the orifice the flashing occurs until, at 15 psi abs back pressure, the flashing begins at the orifice downstream face.

ANALYSIS OF RESULTS

As far as is known, the results presented in this paper and those given by W. T. Bottomley,³ are the only published test data which give the actual flow of saturated water through orifices. In his tests, Bottomley could not, because of limitations in his equipment, determine the effects on the flow caused by (a) varying the back pressure, (b) passing steam with the water, and (c) a static head above or undercooling before the orifice. The results of some of his tests, however, agree very closely with the results found in this investigation as shown in Fig. 2. In accordance with the theory, Bottomley assumed that, even though the actual flow was several times greater than the theoretical, there must be a critical pressure in the orifice; therefore, all of his tests were run with an atmospheric back pressure (14.7 psi abs), which was considered below the actual pressure in the orifice. The data obtained in the present investigation and presented in Fig. 7 do not show the presence of a critical pressure in the orifice, and the pictures in Fig. 11 seem to bear out the conclusion that, for the range of initial pressures included in this study, no critical pressure exists in the orifice. In other words, the change of state does not occur within the orifice.

It is a well-known fact that the weight of steam which will flow through a nozzle is a maximum when the throat pressure is approximately 58 per cent of the upstream pressure. A decrease of

back pressure below 58 per cent of the initial will have no effect on the flow. This, however, is not true for steam flow through a sharp-edged orifice, as is shown by the typical curves⁵ in Fig. 12. Fig. 7 shows a similar orifice characteristic for saturated water, which further substantiates the conclusion that no critical pressure will exist in a sharp-edged orifice passing saturated water, even though the pressure differential across the orifice is sufficient to cause flashing at the downstream face. Whether or not a critical pressure will exist in the throat of a nozzle passing saturated water cannot be determined from the results of this investigation.

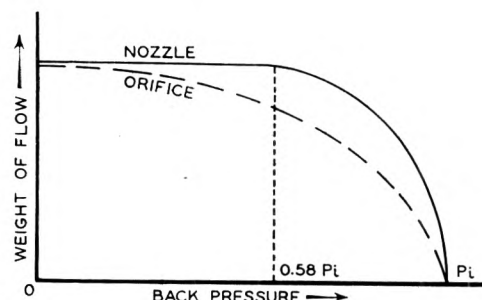


FIG. 12 TYPICAL CURVES SHOWING RELATIVE FLOW OF STEAM THROUGH ORIFICES AND NOZZLES

An orifice, operating between fixed initial and back pressures, will pass a given amount of saturated water according to data in Fig. 7. If the amount of saturated water available is less than the orifice is capable of passing, a small amount of steam will flow through the orifice with the water. The amount of steam which will flow depends upon the initial pressure, the back pressure, and the quantity of water available. For instance, if the initial and back pressures are 145 and 40 psi abs, respectively, and the amount of saturated water to be passed is 2600 lb per sec per sq ft of orifice area, Fig. 9 shows that 0.02 lb of steam will pass through the orifice with each pound of water. From Fig. 8, it is seen that the water flow changes quite rapidly with an increase in steam flow from 0 to 0.04 lb per lb of water; however, for an increase in steam flow above 0.04 lb per lb, the change in water flow is much slower. This fact is also shown in Fig. 9 where the curves crowd together as the steam-flow fraction increases.

The curves shown in Fig. 7 are of the same general shape as the curve giving the flow of "cold" water (70 F) through orifices, which is represented by the equation

$$Q = CA \sqrt{2gh} \dots \dots \dots [1]$$

where Q = the flow in cu ft per sec, A = area in sq ft, h = head in ft of flowing fluid, and C = orifice coefficient. By substituting w for Q and $\frac{144}{\rho} (p_1 - p_2)$ for h , the equation takes the form

$$\frac{w}{A} = \rho C \sqrt{2g \times \frac{144}{\rho} (p_1 - p_2)} \dots \dots \dots [2]$$

in which w = weight of flow in lb per sec, p_1 = initial pressure in psi abs, p_2 = back pressure, v = specific volume of saturated water at p_1 in cu ft per lb, and ρ = density of saturated water at p_1 in lb per cu ft. The equation in this form applies readily to the flow of saturated water through sharp-edged orifices, and values for the orifice coefficient were found to be approximately the same as those for 70 F water. As in the case of cold water, the orifice

⁵ "Thermodynamics," by J. E. Emswiler, First edition, McGraw-Hill Book Company, Inc., New York, N. Y., 1921, p. 225.

"The Leakage of Steam Through Labyrinth Seals," by Adolph Egli, Trans. A.S.M.E., vol. 57, 1935, pp. 115-122.

coefficient decreases with an increase in pressure differential (head) and orifice diameter. There is also some indication that the coefficient decreases as the initial temperature (saturation pressure) increases. The data are not complete enough, however, to make it possible to determine the numerical effect of a variation in orifice diameter or initial temperature. No attempt was made to correlate the orifice coefficients with respect to the diameter ratio since it was thought to be an unwarranted refinement in the design of throttling orifices for which the diameter ratios are usually low. Fig. 13 gives average values of the orifice coefficient for different values of differential head. These are considered sufficiently accurate for many design purposes, without correcting for effect of orifice diameter, initial temperature, or diameter ratio.

Equation [2] and the orifice coefficients given in Fig. 13 may be used also to calculate the flow of saturated water through an orifice when there is a static head. In this case, however, the density of the water depends upon the temperature rather than the pressure at the orifice.

USE OF SINGLE-STAGE ORIFICES FOR DRAINING CONDENSATE FROM FEEDWATER HEATERS

The two important functions performed by a float trap on a feedwater heater are draining the heater and maintaining the

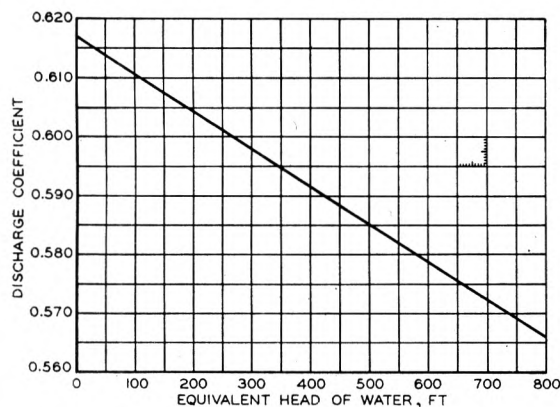


FIG. 13 DISCHARGE COEFFICIENT FOR FLOW OF SATURATED WATER THROUGH SHARP-EDGED ORIFICES

proper pressure differential between the heater and the drain receiver. The float-operated drainer performs these functions by means of a balanced valve, actuated by a float, which responds to changes in hot-well level determined by the quantity of drains. However, these two functions can be performed also by a fixed opening such as a pipe or an orifice and the size of the opening may be varied over a wide range without causing operating difficulty or appreciable thermal loss. For instance, the float-operated trap on a certain heater failed to function at full load on the turbine because of the relatively high pressure drop across the trap valve and, in order to keep the turbine in operation, the heater was drained for several weeks through a 3-in. by-pass around the trap. In this case, since the 3-in. line was several times as large as needed for normal flow, considerable steam cascaded to the next lower pressure heater with the drains. There was, however, no noticeable decrease in heater pressure or cycle efficiency.

The trap on this heater has since been replaced by a $\frac{3}{4}$ -in.-diam single-stage orifice which has been operating successfully for several months. The size of this orifice was determined for full-load condition on the turbine by use of the data given in this paper. At full load practically no steam is passed by the orifice but for two-thirds load, it is estimated that 0.015 lb of steam is passed with each pound of condensate. The method of determining the

orifice diameter and the amount of steam passed at reduced loads is given in detail in the Appendix to this paper.

Whether the turbine cycle efficiency will be affected adversely by cascading some steam between heaters at partial loads depends upon the relative energy drop from the throttle to each extraction point and to the condenser, the method of returning the heater drains to the feedwater circuit, and the relative amount of partial-load operation.

USE OF ORIFICES IN SERIES WITH TRAPS

In cases which require the use of float-operated drainer traps to eliminate thermodynamic losses, due to cascading steam at partial loads, it may happen that an orifice installed on the downstream side of the trap will overcome certain operating difficulties with the trap. In some instances, the large pressure difference between heaters at high turbine loads may cause so much unbalance in the trap valve that the float can no longer operate the valve with the result that the heater floods. When an orifice is installed in series with a trap, the pressure drop between the heaters is divided between the trap and the orifice. By properly designing the orifice, the drop across the trap can be reduced sufficiently to permit the float to operate. A method of designing an orifice to operate in series with a trap is illustrated in Appendix.

While it is possible to use two orifices in series to drain feedwater heaters, it is doubtful whether in most cases they offer any advantages over the single-stage orifice. The design of two orifices to operate in series is much more difficult than the design of a single orifice and, in so far as the authors know, the quantity of steam passed at reduced loads on the turbine can be determined only by a cut-and-try method which is both involved and tedious.

CONCLUSIONS

When saturated water flows through a sharp-edged orifice, no flashing occurs until after the water is through the orifice and, contrary to the theory which is based on a change of state, no critical-pressure condition is evident. The quantity of saturated water that will flow through a sharp-edged orifice for given pressure conditions can be calculated with sufficient accuracy by the formula used to determine the flow of cold (70 F) water through an orifice and the discharge coefficients found for saturated water are approximately the same as those generally used for cold water. When calculating the flow of saturated water through an orifice, it is important to remember that the value of the head to use in the formula is the equivalent head in feet of water, based on the pressure drop across the orifice and the density of the saturated water.

When a mixture of saturated steam and water flows through a sharp-edged orifice with given initial and back pressures a small variation in the amount of steam in the mixture within the range of 0 to 4 per cent has a considerable effect on the total weight of mixture and, consequently, on the weight of saturated water, flowing through the orifice. For mixtures in which the quantity of steam is greater than 4 per cent, a small increase or decrease in steam content has only a slight effect on the total weight of flow.

While sharp-edged thin-plate orifices may be used to drain feedwater heaters in place of float-operated traps, it is probable that for reduced loads some steam will cascade through the orifice with the drains. No general statement can be made at this time concerning the effect on the turbine cycle efficiency of cascading small quantities of steam between heaters. Every case should be decided on its own merits. The necessary study will include consideration of the relative energy drop from the throttle to each extraction point and to the condenser, of the method of returning the heater drains to the feedwater circuit, and of the relative amount of partial-load operation.

It is important to keep in mind that the data presented in this

paper were obtained from tests of sharp-edged thin-plate orifices and, therefore, do not apply to nozzles or short tubes. These data can be used to design a throttling orifice to drain a given amount of saturated or nearly saturated water from a receiver of higher pressure to one of lower pressure and to maintain a required pressure in the former. No attempt should be made to use these data to design a metering orifice. If an orifice discharges into a low-pressure receiver through a pipe, the pressure loss in the discharge pipe must be taken into account in establishing the pressure differential across the orifice. In many cases, the pressure drop across the orifice will be only a fraction of the total drop between receivers, since a large part of the total may be required in getting the flashing mixture of water and steam through the discharge pipe. A discussion of the flow of a mixture of saturated steam and water through pipes is beyond the scope of this study and will be offered in a subsequent paper.

ACKNOWLEDGMENT

The authors gratefully acknowledge the encouragement and help of Messrs. P. W. Thompson, Sabin Crocker, and W. A. Carter in preparing this report and of Messrs. E. L. Liedel and B. Griffin and the technical staff at the Delray Plant in collecting the test data.

Appendix

NOMENCLATURE

The following nomenclature is used in this Appendix:

- A = area of orifice, sq ft
- C = orifice coefficient of discharge
- d = diameter of orifice, in.
- g = acceleration due to gravity, 32 fpsps
- h = static head, ft
- p = pressure, psi abs
- Q = flow of saturated water, cfs
- t = temperature of water, F
- v = specific volume of saturated water, cu ft per lb
- w = flow of saturated water, lb per sec
- ρ = density, lb per cu ft

SINGLE-STAGE ORIFICE DESIGN TO DRAIN FEEDWATER HEATER

When determining the size of an orifice for a particular installation, it is more convenient to use Equation [2] than the data presented in Fig. 7, and it is important to keep in mind that the quantity of condensate to be drained from the heater will vary for a given load on the turbine as much as ± 5 per cent, depending upon the variation in feedwater flow. From an operating standpoint, it is better to design the orifice too large rather than too small, and in practice it is recommended that a hand-operated by-pass be installed to provide means for passing abnormal quantities of water in case of a split or broken heater tube.

$$\text{In Equation [2]} \quad \frac{w}{A} = \rho \times C \sqrt{2g \times \frac{144}{\rho} (p_1 - p_2)}$$

assume the following values corresponding to full-load operation of a 75,000-kw turbine: $w = 13.6$ lb per sec; $p_1 = 236$ psi abs (at saturation temperature); $p_1 - p_2 = 110$ psi; $\rho = 53.8$ lb per cu ft; head = $\frac{144}{\rho} (p_1 - p_2) = \frac{144}{53.8} \times 110 = 294$ ft; $C = 0.598$ (refer to Fig. 13).

$$\text{Therefore} \quad \frac{w}{A} = 53.8 \times 0.598 \sqrt{2g \times 294} = 4450 \text{ lb per sec per sq ft and } A = \frac{13.6}{4450} = 0.00306 \text{ sq ft;}$$

$$d^2 = \frac{4 \times 144 \times 0.00306}{\pi} = 0.561; \quad d = 0.75 \text{ inches}$$

For two-thirds load on the turbine $p_1 = 155$ psi abs; $p_2 = 85$ psi abs; $p_1 - p_2 = 70$ psi; and $\rho = 55.2$ lb per cu ft.

$$\text{Head} = \frac{144}{55.2} \times 70 = 183 \text{ ft; } C = 0.605; \quad \frac{w}{A} = 55.2 \times 0.605$$

$$\sqrt{2g \times 183} = 3620 \text{ lb per sec per sq ft; and } w = 11.1 \text{ lb per sec.}$$

With the initial pressure and pressure differential at this load, the $3/4$ -in. orifice is capable of passing 11.1 lb per sec of condensate. Actually from the heat-balance data, calculated on the basis of no steam flow from the heater, only 7.8 lb per sec of condensate is available; therefore, for the existing pressure conditions some steam will pass through the orifice. Curves A in Fig. 14, which were obtained by cross-plotting the data from Fig. 9 for a constant back pressure of 85 psi abs, show that with an actual flow of condensate of 2540 lb per sec per sq ft for 155 psi abs initial

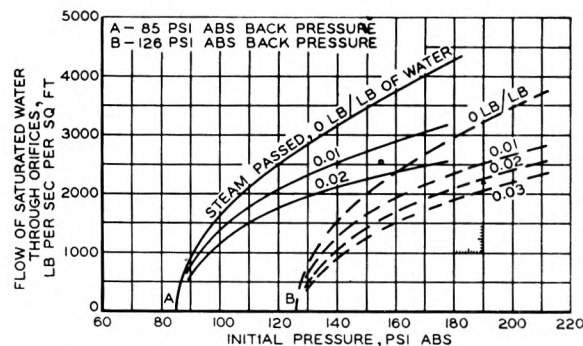


FIG. 14 CURVES ILLUSTRATING METHOD OF DETERMINING SIZE OF ORIFICES FOR SPECIFIED DESIGN CONDITIONS

pressure and 85 psi abs back pressure, approximately 0.015 lb of steam will pass through the orifice with each pound of condensate. After the approximate amount of steam passed through the orifice has been determined, a new heat balance can be made, taking into account the steam cascaded with the condensate.

By comparing the partial-load heat rate of the turbine cycle using orifices with the heat rate at the same reduced loads when only condensate is drained from the heaters, the feasibility of using orifices to drain the heaters for a particular turbine cycle can be ascertained.

DESIGN OF AN ORIFICE TO OPERATE IN SERIES WITH A TRAP

The example just given to illustrate the design of a single-stage orifice for draining a feedwater heater can also be used to show the design of an orifice to operate in series with a trap. In designing an orifice to operate with a trap, the first step is to choose a suitable intermediate pressure which in this case could be 190 psi abs. The following design data are now known: $p_1 = 236$ psi abs; $p_2 = 190$ psi abs; $p_3 = 126$ psi abs; and $w = 13.6$ lb per sec.

The drop in pressure through the trap from 236 to 190 psi abs causes part of the saturated water to flash into steam so that a mixture of 13.3 lb of water and 0.3 lb of steam or 0.0225 lb of steam per lb of water enters the orifice. Curves B in Fig. 14, which were obtained by cross-plotting and extrapolating the data from Fig. 9, show that, for an initial pressure of 190 psi abs and a back pressure of 126 psi abs, an orifice will pass 2200 lb of water per sec per sq ft plus 0.0225 lb of steam per lb of water.

$$\text{or} \quad \frac{w}{A} = 2200 \text{ lb per sec per sq ft; } w = 13.3 \text{ lb per sec}$$

$$\text{and } A = \frac{13.3}{2200} = 0.00605 \text{ sq ft; } d = \sqrt{\frac{4 \times 144 \times 0.00605}{\pi}} = 1.06 \text{ in.}$$

Discussion

F. O. ELLENWOOD.⁶ This paper is of intense interest to the writer and probably to all engineers who are at all concerned with the flow of fluids through orifices. The experimental data presented are valuable to those desiring to use orifices for draining feedwater heaters and also to those who are primarily concerned with the flow phenomena involved.

Even a hasty glance at Fig. 2 of the paper will show that the measured flow of saturated water through a sharp-edged orifice is several times the amount indicated by the so-called "theoretical curve" which is presumably Bottomley's theory. Unfortunately, this theory is not given in the paper, and the reference is not readily available.

It seems to the writer that many engineering papers are somewhat open to criticism when they simply refer to the so-called "theoretical values" without further explanation. In this particular case, the measured rates of flow are undoubtedly correct to a reasonable degree of accuracy, while the Bottomley theory, whatever it may be, is certainly far from complete.

Any theory concerning the flow of a fluid through an orifice, when that fluid is a saturated liquid at entrance, may be made simple or complicated, depending upon how complete it is. If the velocity and density at the orifice could be calculated, it would be a simple matter to determine the rate of flow for any known orifice area. As a matter of fact, however, the velocity and density at the orifice are not simply and accurately calculated. In all cases, the pressure in the orifice will be appreciably greater than that measured at a point considerably beyond the orifice. Just what portion of this total drop in pressure actually occurs in passing through a thin-plate orifice is hard to determine, but it is probably of the order of 25 or 30 per cent. If there is any transformation of liquid into vapor before passing entirely through the orifice, the complete theory then becomes further complicated due to the difficulties of calculating the change in density, the effect of the two-phase velocities, and the energy available to produce velocity in the orifice.

A. E. KITTREDGE.⁷ In commenting on this excellent paper, the writer feels that additional emphasis be placed on the fact that the results refer to and are limited to a thin-plate orifice.

The difference observed between the results of this paper and those of Kittredge and Daugherty⁸ are attributable to the difference between streamline flow and turbulent flow, respectively, as applied to this particular problem. We tested a nozzle subject to turbulent flow. The authors tested a thin-plate orifice subject to streamline flow. Complete turbulent flow represents one limit of the characteristic of the flow of saturated water and complete streamline flow represents the other limit of the characteristic of the flow of saturated water.

As stated, the difference between the results previously obtained and those now observed is a difference resulting from the characteristics of turbulent flow as opposed to the characteristics of streamline flow. In a broad sense the writer believes this to be true but there are other elements, namely, time, mass, energy, and heat-transfer rate which very likely influence the existence or nonexistence of a critical pressure in the flow of saturated water. In contrast to the flow of an almost perfectly elastic gas or vapor, allowance must be made for the fact that saturated water is initially a much denser fluid; that the ratio of the specific volume at critical pressure to the initial specific volume is much

greater for saturated water than for any ordinary gas or vapor; that the flashing of water involves a complete change of state, not just a readjustment of the pressure-volume relation; that the change in state involves heat transfer from the mass surrounding a bubble to the bubble formed; and that more time and more energy per unit volume of fluid are required to redistribute the mass of saturated water from its initial volume to its greatly enlarged mixed volume after flashing, as compared to the time and energy required per unit volume to redistribute the mass of a relatively light and nearly perfectly elastic gas or vapor from its initial volume to its moderately increased volume at its critical pressure.

Turbulence and time contribute to all of these factors involved in the change of state between the initial pressure and the critical pressure. It is quite possible that, in the case of a thin-plate orifice, the completion of the change in state requires more time than elapses from the point of initial acceleration to the vena contracta of the orifice. In the authors' experiment, orifice sizes, ranging from $1/4$ in. diam to $7/8$ in. diam were used. Liquid velocities involved were of the order of 100 fps. If, in the case of a $1/4$ -in.-diam orifice, the acceleration of the liquid occurred in a distance of $1/4$ in. the time available for changing state during the period of acceleration and pressure reduction was approximately 0.0002 sec.

In February, 1934, in addition to the $1/4$ -in.-diam nozzle just described, we tested the application of our theory to a $1/2$ -in.-diam standard iron pipe about 10 ft long. The existence of critical pressure was determined by locating a pressure gage on the pipe at the discharge end. The results verified logical theory. This particular test setup certainly provided turbulent flow. The writer would suggest that a worth-while research would be to establish the relation between hydraulic diameter and length of pipe line or nozzle required to develop full turbulence and full flashing in accordance with thermodynamic theory.

The specific purpose of the present paper has been to determine flow rate through thin-plate orifices, but the inspiration for the investigation and the broad purpose of the paper is to determine the possibility of using orifices for the drainage of stage heaters. In this regard, it is felt that the authors have missed a remarkable opportunity by failing to investigate the characteristics of turbulent flow. For the particular purpose of draining stage heaters, a turbulent nozzle, having a capacity with saturated water substantially directly proportional to the absolute pressure, is much preferable to a thin-plate orifice, having a capacity varying as the square root of the absolute pressure. At the same time, the turbulent nozzle would have much lower capacity under saturated water and much wider range of control due to static head on the inlet side. If operating engineers are determined to eliminate interstage traps, they should attack the proposition on the basis of a turbulent nozzle located a considerable distance below the heater to be drained so that it may be subject to appreciable submergence on the inlet side. Submergence on the outlet side does not matter since flashing of the liquid will so reduce the density on the outlet side that, with reasonable pipe sizes, the pressure on the discharge side of the turbulent nozzle will never be higher than the critical pressure. Apparently then, the ideal automatic drainage arrangement for stage heaters would consist of a U-seal arrangement with the two legs of about the same diameter and the crossover connection between the two legs consisting of a small-diameter tube developing turbulent flow. The crossover tube between the two legs of the U-tube connection might be not less than one half the diameter of the respective leg.

The concluding paragraphs of the paper under discussion are rather vague regarding the applicability of the thin-plate orifice to the service of interstage draining of surface heaters but sug-

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⁷ Chief Engineer, Cochrane Corporation, Philadelphia, Pa.

⁸ "Discharge Capacity of Traps," by A. E. Kittredge and E. F. Daugherty, *Combustion*, vol. 6, September, 1934, pp. 14-19.

gests the possible use of a trap followed by an orifice. The writer would raise the question: If a trap is necessary why use the orifice? The trap in a full-open position is an orifice. This question is asked with particular reference to new installations and does not contemplate the possible expediency of assisting a defective trap.

S. P. SOLING.⁹ The authors have shown that, in the case of water, the actual flow of saturated liquid through orifices is greater than would be expected from theoretical considerations. Their graphical treatment clearly indicates how this occurs.

The same behavior has been noted for the refrigerants dichlorodifluoromethane (Freon-12) and ammonia. Orifices sized on the basis of equilibrium conditions in the orifice proved several times too large. Quantitative results are not available for comparison, as various amounts of vapors were passed with the liquid for different runs, the orifice readings being incidental to a test. No attempt was made to obtain data as comprehensive as those of the authors' who are to be congratulated on their clarification of a puzzling problem.

D. R. YARNALL.¹⁰ In connection with this paper, it might be interesting to compare the conclusions with the trend of data obtained in the study of the flow of saturated and subcooled water through a rounded entrance orifice. This work was undertaken in the fall and winter of 1938-1939 by the research department of the writer's company and utilized a setup somewhat similar to that of the authors'. This orifice was connected to the mud-drum blowoff of a small high-pressure test boiler, using distilled feedwater.

The orifice used was 0.130 in. diam with a rounded approach of $\frac{1}{8}$ in. radius, followed by a tubular section $\frac{1}{8}$ in. long. A small pressure tap was drilled radially into the throat of the orifice to

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¹⁰ Research Department, Yarnall-Waring Company, Philadelphia, Pa. Fellow A.S.M.E.

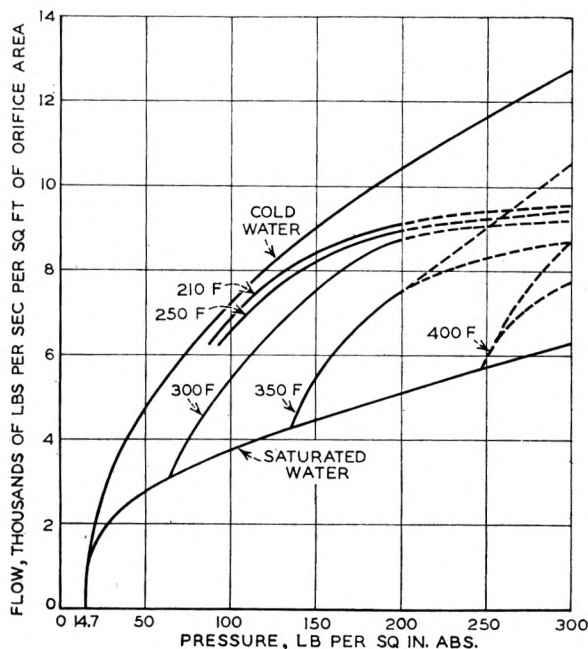


FIG. 15 FLOW CURVES FOR SATURATED AND SUBCOOLED WATER, BASED ON ACTUAL FLOW THROUGH A ROUNDED ORIFICE

(Orifice, 0.130 in. diam, rounded to $\frac{1}{8}$ in. radius, and with throat, $\frac{1}{8}$ in. long.)

obtain some measurement of pressure conditions existing in the throat.

The results obtained were in general consistent with the trend of flow values disclosed in the paper, being greatly in excess of flow calculations based on thermodynamic equilibrium. From practical equality with cold-water flow at low heads, the ratio of the flow of saturated-water to cold-water flow reduced to approximately 0.6 at 50 psi abs; and further to approximately 0.5 at 100 psi abs, where the ratio to cold-water flow remained essentially constant up to 300 psi abs, the highest pressure reached. The actual numerical flow values obtained for saturated-water flow through the rounded orifice (in lb per sec per sq ft of orifice area), check within close limits of those given in the paper. However, when adjustment is made for differing coefficients, the flow through the rounded orifice will be considerably less in comparison.

On subcooled water, isothermal-flow curves, Fig. 15 of this discussion, showed a tendency to parallel the cold-water-capacity curve at pressures up to 200 psi abs. At higher pressures (between 200 and 300 psi abs) inconsistent results were obtained, the majority of observations indicating a marked drop in the rate of increase in flow with increased pressure. Due to capacity limitations of the apparatus and interruption of the investigation, this trend has not been conclusively substantiated to date. It would be interesting to know whether there was any indication pointing toward reduced capacity on subcooled water flowing through a sharp-edged orifice at pressures above 200 psi abs?

It is gratifying to note the growing consistency of accumulating data on the flow of saturated water and it is particularly helpful from the practical standpoint to find a simplified approach to a phase of the subject such as the authors of this paper have so clearly outlined. It is expected that work on the flow through rounded entrance orifices will be resumed, concluded, and the results submitted for publication shortly, in order that these findings may also contribute to the general subject matter.

AUTHORS' CLOSURE

The authors are extremely grateful for the interest shown by those who prepared discussions of their paper and feel that each discussor has added to the value of the paper by presenting his comments. The authors had hoped, however, that more interest would be shown in the possible application of these data in the design of orifices for specific uses, such as that of draining extraction feedwater heaters as described in the paper. The authors would like to emphasize again the importance of that part of their paper on the assumption that this point may have been missed by many who are interested in design work involving the flow of saturated liquids.

Regarding Professor Ellenwood's criticism of the paper because the theoretical treatment was omitted, it should be remembered that the authors were required to meet certain space limitations. The theory given in Bottomley's paper,⁸ which was referred to by the authors, merely assumes thermodynamic equilibrium in the orifice. We realize, and regret, that Bottomley's paper is not widely distributed, but it is available in the Engineering Societies' Library, New York, N. Y. As usual, Professor Ellenwood's comments are welcome, and it is hoped he will be encouraged to continue the theoretical study of the phenomena of the flow of a saturated liquid through orifices and ultimately report his conclusions.

It is not clear to the authors what Mr. Kittredge means by "streamline" and "turbulent" flow in reference to this paper. Based on the usual Reynolds' number criterion, the flows reported are definitely in the turbulent region. If Mr. Kittredge's use of the term "turbulent" refers to a condition of flashing such as might occur in the downstream portion of a nozzle, or in the tail pipe immediately following an orifice, then the authors agree that

the results of the orifice tests would not apply. The paper specifically states that the orifice results do not apply to nozzles or short tubes. In view of their test results, however, the authors are at a loss to know how a flashing mixture of steam and water is to be obtained in a sharp-edged thin-plate orifice supplied only with saturated water. As shown by the results in the paper, if steam is passed through the orifice with the water, the capacity of the orifice is considerably less than if water alone is passed, however, this mixture is not the result of flashing in the orifice.

Mr. Kittredge's comments on several of the differences between saturated water and an elastic gas are pertinent to a theoretical analysis of the problem and add considerably to the background of the discussions. In addition to these theoretical factors, it probably is true that surface tension also plays an important part in retarding the flashing of a saturated liquid. This factor was pointed out by Prof. M. C. Stuart and others in oral discussion.

The authors are grateful to Mr. Kittredge for emphasizing the underlying purpose of the investigation. We feel that the practical applications of thin-plate orifices deserve serious consideration by designing and operating engineers. However, regarding the charge that "a remarkable opportunity had been missed by failing to investigate the characteristics of turbulent flow," the authors would like to say that an investigation of the flow of a flashing mixture of water and steam through pipes has been made and the results will be offered for publication later. These results were not included in the present paper because of lack of space and because the authors felt that each phase of this subject deserved the emphasis derived from a separate report.

Mr. Kittredge's suggestion of using a U-leg arrangement for draining feedwater heaters is basically sound, but the authors point out that one of the primary reasons for using orifices for draining heaters is that it is possible to install an orifice at the end of the cascade drain line and thus prevent erosion of elbows resulting from high velocities. This was mentioned in the "Introduction" of the paper as one of the important advantages of orifices over traps, and it is also one of the main advantages of the former over Mr. Kittredge's U-leg arrangement.

In answer to Mr. Kittredge's question: "If a trap is necessary why use the orifice?" it is obvious from his own qualifying statement following his question that he already knows the answer. As is well known to plant operators and trap manufacturers alike,

there is nothing uncommon about the failure of a supposedly non-defective trap to function properly under some operating conditions even though it was "designed for the job." Where a trap needs assistance because the float is too small to overcome the unbalance in the trap valve, the use of an orifice in series is a simple, inexpensive expedient.

Mr. Soling's comments are most welcome in that they point out that saturated liquids other than water behave in the same general way as water when flowing through an orifice or valve into a region in which the pressure is lower than the saturation pressure.

Mr. Yarnall is to be commended for publishing some of his test data. These data supply some of the information on the characteristics of flow of saturated water through a nozzle or short tube which were lacking in the authors' results and, in this respect, his discussion is a distinct contribution. The authors do not understand why the bellmouth tube should have a relatively smaller capacity than a sharp-edged orifice, after correcting for differences in the cold-water-discharge coefficients. It may be that the relatively small size of tube used in Mr. Yarnall's tests is responsible for the difference, either because of undisclosed factors associated with the small physical size or because of difficulties involved in making the laboratory determinations. The latter possibility is indicated by the inconsistent results at high pressure differences. The marked drop in rate of increase of flow with increased pressure differential may indicate a critical pressure condition in the tube. This may be what Mr. Kittredge refers to as a turbulent condition. It would seem to the authors, however, that this drop in the rate of increase of flow would have been more pronounced with saturated water than with subcooled water, which is contrary to the indications of Fig. 15 of Mr. Yarnall's discussion. In the authors' tests the reduction in discharge coefficient for increasing pressure differentials with subcooled water was essentially the same as shown in Fig. 13 for saturated water.

A recent communication from Prof. J. I. Yellott makes an interesting analogy between the so-called supersaturation in a rapidly expanding steam jet and the failure to flash in the saturated water jet. He says, "In a very rapid expansion, a substance in the liquid or vapor phase is apparently unable to change its phase rapidly enough to alter the flow characteristics of nozzles or orifices."

Train Acceleration With Steam Locomotives

By L. B. JONES,¹ ALTOONA, PA.

This paper presents a study of the relation between the cylinder tractive force of high-speed steam locomotives and the energy required to accelerate trains of differing weights at various rates. Mathematical formulas for computing time and distance required for acceleration are presented in an Appendix. Consideration is given to some of the more important factors which limit cylinder tractive power.

THE mathematics of acceleration of railway vehicles has been fully discussed in various textbooks and also in previous papers presented before the Society; but for convenience of ready reference the fundamental concepts are herein reviewed. Weights of locomotive and train are expressed in tons; and acceleration is expressed in terms of miles per hour per minute or per mile, to conform with the customary statistics of train schedules; in distinction to the common formulas of physics and mechanics expressed in terms of pounds, feet, and seconds. By this means, it is hoped to record data which will be helpful to operating officers as well as to designing engineers. Because the energy of acceleration varies with the square of the velocity, but only directly with the weight of the train, simple arithmetical proportion is deficient when comparing different locomotives or different weights of trains at the higher speeds, and a graphic analysis is most useful to show what actually takes place.

The force available for acceleration in the cylinders of the ordinary two-cylinder steam locomotive is expressed by the well-known formula

$$T = \frac{C^2PS}{D}$$

where T = cylinder tractive effort, lb
 C = mean diameter of the cylinders, in.
 P = mean effective pressure, psi
 S = piston stroke, in.
 D = diameter of drivers, in.

The formula contains three fixed dimensional values and only one value subject to variation with speed, i.e., the mean effective pressure. It therefore follows that, as this value is maintained or increased, the cylinder tractive force will be maintained or increased; which is the only force, on level track, available to accelerate the train. Therefore, the ability of a steam locomotive to accelerate a train rests with its mean effective pressure.

In Figs. 1 and 2 are shown cylinder-horsepower and cylinder-tractive-force versus speed curves for several locomotives, in which curve A represents a Pacific-type locomotive which has been performing satisfactorily in main-line passenger service for several years. For purpose of this study, curves B , C , D , and E represent successive improvements in the mean effective pressure of this same locomotive, but for simplification the studies of train acceleration are confined to the minimum or present locomotive A , and the maximum or improved locomotive E . The latter has been selected as the maximum locomotive for this study because,

as shown in Fig. 1, the cylinder horsepower is maintained almost constant from 60 to 100 mph. While it is sufficiently in advance of current steam-locomotive practice to be called a "maximum" locomotive, it is by no means an "ultimate" locomotive because, if the mean effective pressure could be still better maintained as the speed increases, the horsepower would actually increase with the speed above 60 mph, as it now does below that speed.

If yet greater power must be obtained, a glance at the formula previously given will show that the only recourse is redesign, or increase of one or more of the dimensional values. The advantages of improving the present locomotive, as compared with design changes, involving increased weight and capital investment, are illustrated by the curves in Figs. 4 to 8, inclusive, which have been developed on the assumption that improved locomotive E has been produced from present locomotive A without any increase in weight.

For this study, three trains weighing respectively 800, 1000, and 1200 tons behind the tender have been assumed, and their gross resistances, based on the Davis formulas, are shown in Fig. 3. For simplicity, all calculations have been based on straight level track; the effect of grades, plus or minus, may be added or subtracted, and a similar correction may be made for curves. For purposes of comparing two or more locomotives, the assumption

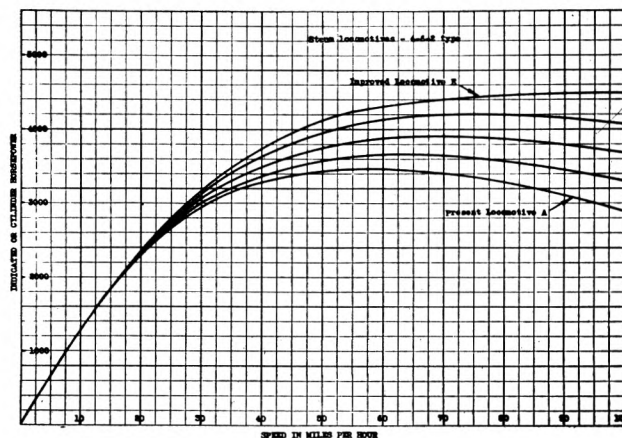


FIG. 1 SPEED AND HORSEPOWER CURVES

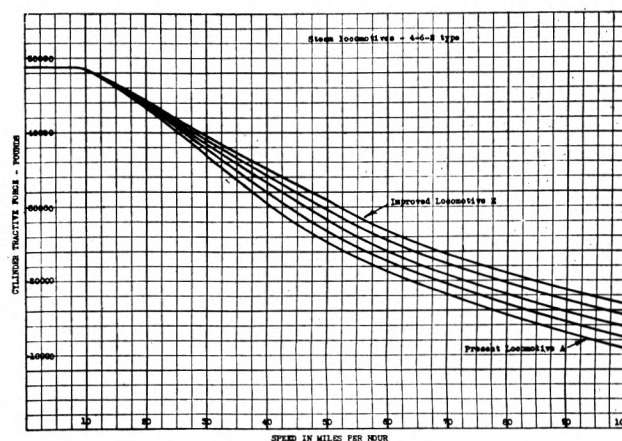


FIG. 2 SPEED AND TRACTIVE FORCE CURVES

¹ Engineer of Tests, The Pennsylvania Railroad. Mem. A.S.M.E. Contributed by the Railroad Division and presented at the Annual Meeting, New York, N. Y., December 2-6, 1940, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

of level track will answer as well as any other condition which might be selected.

The curves involving speed, time, and distance were calculated from the tractive-force and resistance curves point by point and then verified by the mathematical formulas presented in the Appendix. In each case, the two methods checked very closely; and it is evident that acceleration curves can be constructed by the formulas which will reflect the effect of changes in the tractive-power curve on the performance of the locomotive. Since

the curves are plotted for the minimum and maximum locomotives only, it is also evident that the performance of the intermediate locomotives, *B*, *C*, and *D*, can be studied from the curves by interpolation. It will be noted that the mathematical studies in the Appendix follow closely the methods of Professor Barrow (1).²

Fig. 4 compares present locomotive *A* with maximum loco-

² Numbers in parentheses refer to the Bibliography at the end of the paper.

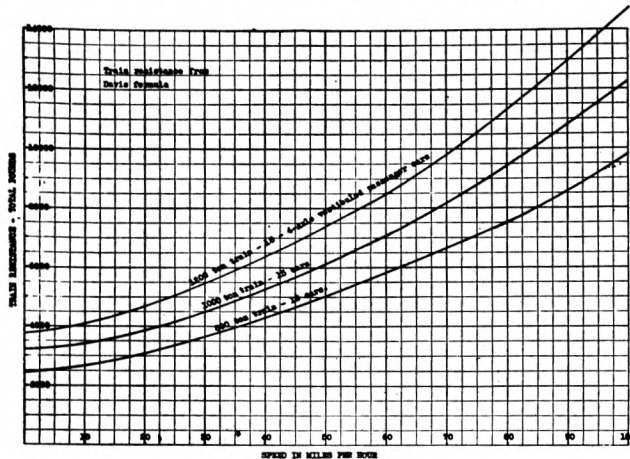


FIG. 3 SPEED AND TRAIN-RESISTANCE CURVES; LEVEL TRACK

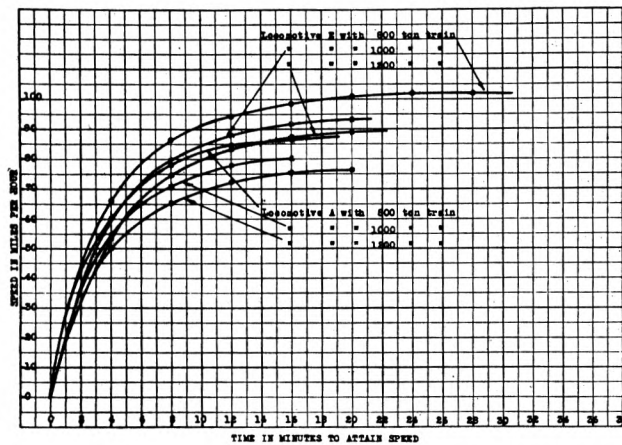


FIG. 4 TIME AND SPEED CURVES

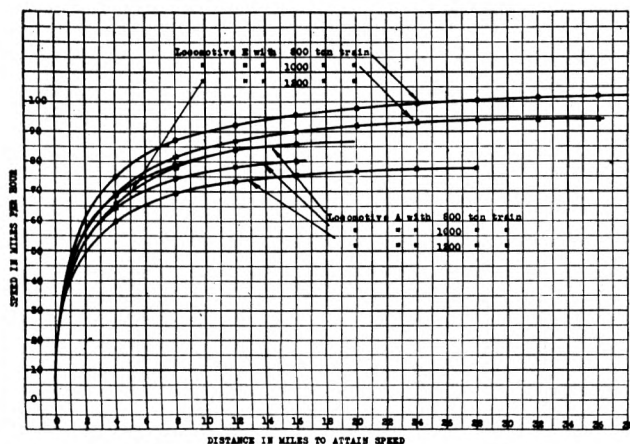


FIG. 5 DISTANCE AND SPEED CURVES

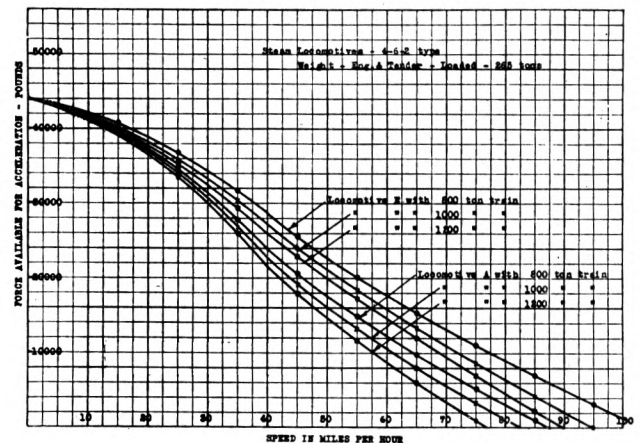


FIG. 6 SPEED AND ACCELERATING-FORCE CURVES

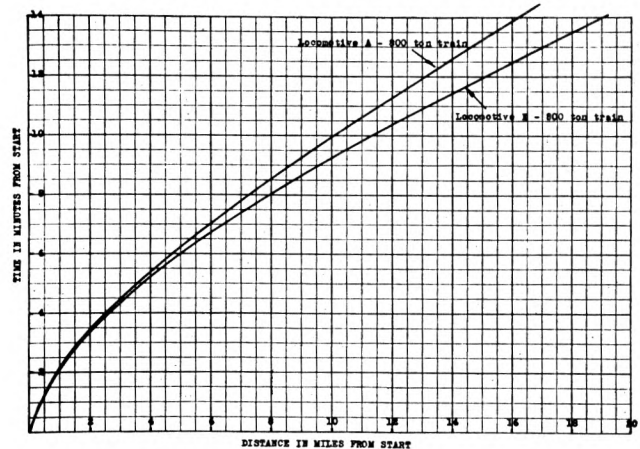


FIG. 7 TIME AND DISTANCE CURVES

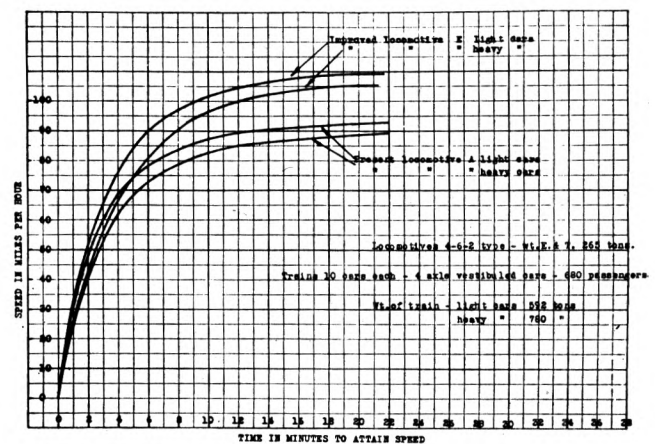


FIG. 8 LIGHT CARS VERSUS HEAVY CARS

tive *E*, hauling three different trains, and Fig. 5 illustrates the same comparison based on distance. It will be noted that the higher sustained horsepower of the improved locomotive results in a reduction of both time and distance required to attain a given speed.

These curves also serve to emphasize a point which must be borne in mind by operating officers, and that is the serious handicap of enforced slowdowns. Locomotive *A* with an 800-ton train requires approximately $2\frac{1}{2}$ min or $1\frac{1}{2}$ miles to attain 50 mph, and $6\frac{1}{2}$ additional min or $7\frac{1}{2}$ miles to attain 80 mph; so that, if the train is slowed from 80 to 50 mph, $6\frac{1}{2}$ min or $7\frac{1}{2}$ miles are required to resume the original speed. This should not be confused with elapsed time, which would include also time lost in slowing down and running at reduced speed, items not covered by this investigation.

Fig. 6 shows the force available for acceleration compared with speed. The locomotive has reached its maximum speed when the accelerating force becomes zero. To determine the maximum speed of the locomotives on a grade, it is only necessary to determine the grade resistance of the locomotive and train and draw a horizontal line at the corresponding value. The intersection of the curves with the line so drawn will show the maximum speed on the grade selected.

Fig. 7 shows a mathematical "race" between locomotives *A* and *E*. Starting from the same point, with trains of identical weight, it will be seen that, at the end of 12 min, locomotive *E* is 2 miles ahead of locomotive *A*; and the gap widens rapidly due to the more rapid acceleration of the improved locomotive.

Fig. 8 shows the effect of lightweight cars on the rate of acceleration. The weights of the two trains, with a given locomotive, are proportional to the time required to attain the same speed, and to the squares of the speeds attained in a given time. Therefore, it follows that, for a given maximum speed, the saving in schedule time by the lightweight train is confined to acceleration, and if there are no stops or speed reductions, the heavy train will require only a little more time to cover a given distance than the lightweight train. On the other hand, if there are numerous stops and slowdowns, the advantage of the lightweight train is multiplied.

Fig. 9 shows tractive-force curves for steam, electric, and Diesel locomotives of equivalent-nominal-horsepower rating. Steam locomotive *E* from previous studies is compared with assumed electric and Diesel locomotives, the continuous motor rating and the Diesel-engine rating being used for the electric and Diesel locomotives, respectively. It is recognized to be common practice to take advantage of the overload capacity of electric motors while accelerating, which is a distinct advantage for an electric locomotive drawing its power from a trolley; but the Diesel is limited by the capacity of its engine, and the overload possibilities of the steam locomotive are circumscribed by considerations of economy and good practice, at least in the preparation of train schedules. A direct comparison of locomotives having such different characteristics is impossible, but the curves serve to illustrate the relative capacities for accelerating trains. They also demonstrate that the steam locomotive, with moderate improvement, is capable of taking rank with the best motive-power units.

Fig. 10 illustrates an advantage of the improved locomotive with respect to the power output required for acceleration to a given speed. The kinetic energy of two trains of the same weight is the same for any speed; but the improved locomotive requires less time and distance to attain speed and, therefore, the energy required to overcome friction is less. Inasmuch as each locomotive would have to cover the same distance in actual operation, this saving during acceleration is theoretical rather than real.

Perhaps the greatest handicap of the steam locomotive is the

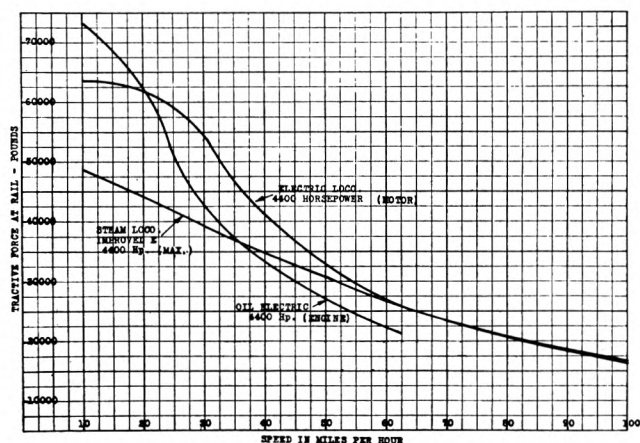


FIG. 9 TRACTIVE FORCE OF VARIOUS LOCOMOTIVES

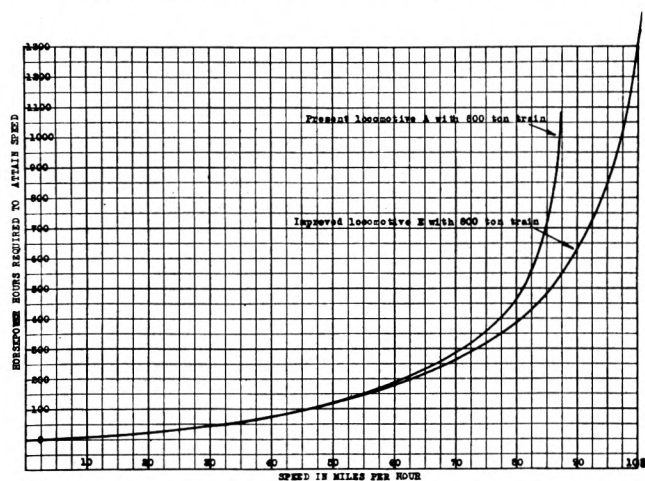


FIG. 10 SPEED AND HORSEPOWER HOURS

deep-rooted conservatism of American locomotive designers which has sentenced it to be a machine of two cylinders controlled by one valve apiece. The destructive dynamic forces which diverge from the line of power transmission, and the ineffective steam distribution have, it seems, become necessary evils, to be tolerated rather than faced. The author believes that these handicaps can be overcome, while still retaining for the steam locomotive the simplicity and flexibility which are its greatest assets. The maximum locomotive *E* has been assumed as a two-cylinder locomotive, conventional in all respects but valve action; and it seems probable that no satisfactory arrangement of cylinders to eliminate counterbalances and dynamic augments can be developed until a satisfactory valve action has been perfected. But regardless of the number and arrangement of the cylinders, the mean effective pressure will continue to govern the output; and the following additional assumptions have been made:

(a) Minimum pressure drop from boiler to steam chest. The superheater and pipes should afford free passage to the steam, for while steam which expands without doing work is raised in temperature, it is pressure which does the work in the cylinders.

(b) Adequate steam-chest volume. The opening of the admission valve results in equalization of pressures in the steam chest and cylinder; and at high speeds the surge of steam pressure from the pipes and header does not reach the steam chest until the valve has closed. The result is a maximum indicator-card pressure far below boiler pressure, and an average admission pressure yet lower. Meantime, the steam entering the chest at high

velocity builds up a surge pressure which may go 50 lb above boiler pressure at its peak, but drops to normal before the next valve opening. An adequate steam-chest volume is, therefore, essential to hold up the admission line on the indicator card; and also to insure a uniform velocity of steam through the superheater and pipes.

(c) Well-designed exhaust passages. The ideal passage would pass the steam to the nozzle at maximum velocity and minimum back pressure; but since both are impossible of attainment in the same passage, a uniform cross section of smooth proportions is desirable. Too large an exhaust passage operates as an expansion chamber which has to be choked at the nozzle to produce draft, with resulting high back pressure against the piston in the center of its stroke, where it is most damaging to the mean effective pressure.

(d) Large exhaust nozzle, which is only possible with an efficient front end.

(e) Proper steam distribution. This specification eliminates the one-piece reciprocating valve, and requires separate admission and exhaust valves so arranged that cutoff may be shortened without advancing the other valve events. Various valve arrangements which meet this requirement more or less perfectly are extensively used in Europe and we would do well to profit by their experience. Experiments now under way in this country may lead to successful results.

Appendix

EQUATIONS FOR ACCELERATING FORCE

When the tractive-force-speed curve of a locomotive is known, the curve for train resistance of any given train can be subtracted and the result is the accelerating-force-speed curve for the combined locomotive and train.

Let F = accelerating force for entire train, lb
 a = acceleration, mphs = 1.467 fpsps
 W = weight of train, including additional percentage to provide for energy of rotation, tons
 V = speed, mph
 L = distance traveled to reach any speed, miles
 M = mass of train = $\frac{W \times 2000}{32.2}$
 t = time to reach any speed, min
 $A, B, C, D, K_1, K_2,$ and K_3 are constants

Since $a = \frac{F}{M \times 1.467}$

$$a = \frac{F \times 32.2}{2000 \times W \times 1.467} \text{ mphs}$$

or $a = \frac{21.95F}{2000W}$ also, $a = \frac{dv}{dt}$

$$\frac{dv}{dt} = \frac{21.95F}{2000W} \text{ or } dt = \frac{2000W}{21.95F} dv$$

$$dt = \frac{91.1Wdv}{F}$$

The time, t to reach any speed is

$$t = \frac{91.1W}{60} \int \frac{dv}{F} \dots \dots \dots [1]$$

The distance L traveled in miles to reach any speed is $dL =$

$K_1 V dt$ where K_1 , is the constant required to make the relation true.

$$K_1 = \frac{1.467}{5280} \quad \text{so} \quad dL = \frac{1.467}{5280} V dt$$

$$dL = \frac{1.467 \times 91.1WVdv}{5280F} = 0.02533 \frac{WVdv}{F}$$

or

$$L = 0.02533W \int \frac{Vdv}{F} \dots \dots \dots [2]$$

The constant of integration required to make the equation true for a known condition will be called D .

Knowing the relations shown by Equations [1] and [2], it only remains to write the equation for speed V and tractive force F for the train, from which equations showing the time and distance to reach any speed can be derived.

The force of acceleration is maximum at starting. As long as the engine can be run in full gear, the accelerating force falls off from its original value by an amount which is proportional to the square of the speed. The general equation for this relationship is

$$F = F_0 - K_2 V^2 \dots \dots \dots [3]$$

where F_0 equals starting tractive force and K_2 = constant required for any particular curve. This condition holds for a passenger train until a speed of about 30 mph is reached, following which, changed cutoff and other conditions give quite a different curve. The curve changes from convex to concave in the 30-mph speed range (sometimes referred to as the critical range). Therefore, two equations are required for every speed-accelerating-force curve.

To simplify the actual calculations, Equation [3] was not used for the accelerating-force-speed curves but the equation was changed to

$$F = \frac{F_0}{1 + K_3 V^2} \dots \dots \dots [4]$$

It is obvious that the substitution of the value of F in Equations [1] and [2], as given by Equation [4], is much more easily handled than the substitution which Equation [3] gives. The substitution of Equation [4] in Equation [1] gives

$$t = \frac{91.1W}{60F_0} \int (1 + K_3 V^2) dV$$

which can be very quickly solved.

The substitution of Equation [3] in Equation [1] gives

$$t = \frac{91.1W}{60} \int \frac{dV}{F_0 - K_2 V^2}$$

which is much less convenient.

When constants K_2 and K_3 are chosen so that Equations [3] and [4] match at 0 and 30 mph, the intermediate points are close enough to give the correct time and distance for all speeds.

As an example, the following table shows how Equations [3] and [4] compare, for a case where the accelerating force equals 44,000 lb at starting and 36,000 lb at 30 mph. For Equation [3] the equation is

$$F = 44,000 - 8.88V^2, \text{ or } K_2 = 8.88$$

for Equation [4]

$$F = \frac{44,000}{1 + 0.0002468V^2}, \text{ or } K_3 = 0.0002468$$

Speed	Accelerating force, lb	
	Equation [1]	Equation [2]
0	44000	44000
10	43120	43000
20	40480	40050
30	36000	36000
35	33100	33550

It is apparent that for all practical purposes the two equations are equivalent, so the one most readily usable should be chosen.

The curve for the higher speeds, which is the more important, is found by two steps. The first step is to note that if a tangent TT_1 to the tractive-force-speed curve SS_1 is drawn, the difference in ordinates of the tangent and the curve, when plotted on rectangular coordinates, forms a very good parabola, Fig. 11. The next step is to write the equation of this parabola. When the equations of the tangent and parabola are added, the result is the equation of the tractive-force curve. This curve always has the form

$$AV^2 + BV + C = F \dots \dots \dots [5]$$

In most cases, where a well-chosen point of tangency is used, the curve of Equation [5] fits the actual curve very closely. Almost any point of tangency gives good results.

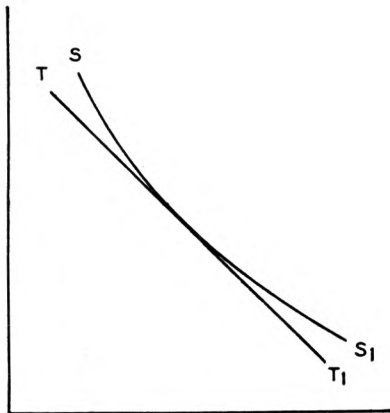


FIG. 11

Substituting the values of F in Equations [4] and [5] in Equations [1] and [2] gives

$$t = \frac{91.1W}{60F_0} \int (1 + K_3 V^2) dV$$

$$t = \frac{1.5166W}{F_0} \left(V + \frac{K_3 V^3}{3} \right) \dots \dots \dots [6]$$

which holds good to 35 mph, above which

$$t = \frac{91.1W}{60} \int \frac{dv}{AV^2 + BV + C}$$

$$t = \frac{91.1W}{60} \frac{2.303}{\sqrt{B^2 - 4AC}} \log \left(\frac{2AV + B - \sqrt{B^2 - 4AC}}{2AV + B + \sqrt{B^2 - 4AC}} \right) + D \dots [7]$$

which holds good above 30 mph, and allows a short overlap with the curve of Equation [6] in the critical range.

The time t is calculated by Equation [6] for the lower speeds; then D is calculated to make the time t correct at 35 mph in Equation [7].

For distance L

$$L = 0.02533W \int \frac{Vdv (1 + K_3 V^2)}{F_0}$$

from Equations [2] and [4]

$$L = \frac{0.02533W}{F_0} \int V(1 + K_3 V^2) dv$$

$$L = \frac{0.02533W}{F_0} \left(\frac{V^2}{2} + \frac{K_3 V^4}{4} \right) \dots \dots \dots [8]$$

which is good up to 35 mph. For high speeds

$$L = 0.02533W \int \frac{Vdv}{AV^2 + BV + C}$$

$$L = \frac{2.303 \times 0.02533W}{2A} \left(\log (AV^2 + BV + C) - \frac{B}{\sqrt{B^2 - 4AC}} \log \frac{2AV + B - \sqrt{B^2 - 4AC}}{2AV + B + \sqrt{B^2 - 4AC}} \right) + D$$

It is noted that $AV^2 + BV + C = F$ at any speed so F_v can be substituted for $AV^2 + BV + C$ in the foregoing equation which gives

$$L = \frac{2.303 \times 0.02533W}{2A} \left(\log F_v - \frac{B}{\sqrt{B^2 - 4AC}} \log \frac{2AV + B - \sqrt{B^2 - 4AC}}{2AV + B + \sqrt{B^2 - 4AC}} \right) + D \dots [9]$$

where D makes L correct for 35 mph, according to Equation [8].

Equations [7] and [9] would not be applicable if the quantity B^2 did not exceed $4AC$, but B^2 must always exceed $4AC$, otherwise no speed could be reached where there would not be some accelerating force. The maximum speed is found when

$$F = 0 \text{ or } AV^2 + BV + C = 0$$

$$\text{then } V = \frac{2C}{\sqrt{(B^2 - 4AC)} - B} \dots \dots \dots [10]$$

Inspection of Equation [10] shows that if $4AC$ were greater than B^2 , the maximum speed would have no limit.

The application of Equations [7] and [9] is quite simple because the quantity $\sqrt{B^2 - 4AC}$ which is easily computed repeats itself so frequently. The quantity $(AV^2 + BV + C)$ is really F_v at any speed, and the quantities $B \pm \sqrt{B^2 - 4AC}$ for a large group of curves can be conveniently arranged in a table. Then to obtain Equations [7] and [9], it is simply a matter of substituting, and calculating the particular constants of integration, which give the correct time and distance at 35 mph, as obtained from the more elementary Equations [6] and [8].

Ten cases were solved along the foregoing lines. They were for trains hauled by the present passenger locomotive *A* and the improved passenger locomotive *E*, using 800-, 1000- and 1200-ton trains, a loaded 10-car passenger train made up of heavyweight cars, and a loaded 10-car passenger train made up of lightweight cars.

The following table shows the make-up of the trains:

Condition	No. of cars	Weight of cars, tons	Gross weight of train, tons	(W) weight, allowing for kinetic energy of wheels and axles, tons
1	12	800	1065	1098
2	15	1000	1265	1303
3	18	1200	1465	1507
4	10	592	858	891
5	10	780	1045	1077

The equations for the various curves may be listed as follows:

Case 1, condition No. 1; present locomotive A:

Below 35 mph

$$F = \frac{44000}{1 + 0.00045V^2}$$

$$t = 0.03785V + 0.00000568V^3$$

$$L = 0.000316V^2 + 0.000000071V^4$$

Above 30 mph

$$F = 2.9V^2 - 869V + 54000$$

$$t = 10.66 \log \frac{1228 - 5.8V}{510 - 5.8V} - 4.014$$

$$L = 11.02 \log F_v + 26.68 \log \frac{1228 - 5.8V}{510 - 5.8V} - 62.34$$

Case 2 Condition No. 2; present locomotive A:

Below 35 mph

$$F = \frac{44000}{1 + 0.000455V^2}$$

$$t = 0.045V + 0.00000684V^3$$

$$L = 0.000375V^2 + 0.000000085V^4$$

Above 30 mph

$$F = 4.21V^2 - 1069.5V + 59530$$

$$t = 12.07 \log \frac{1447 - 8.42V}{692 - 8.42V} - 3.68$$

$$L = 9.017 \log F_v + 25.55 \log \frac{1447 - 8.42V}{692 - 8.42V} - 51.22$$

Case 3 Condition No. 1; improved locomotive E:

Below 35 mph

$$F = \frac{44000}{1 + 0.00031V^2}$$

$$t = 0.03785V + 0.0000039V^3$$

$$L = 0.000316V^2 + 0.000000049V^4$$

Above 30 mph

$$F = 3V^2 - 887V + 59350$$

$$t = 14.06 \log \frac{1160 - 6V}{614 - 6V} - 3.72$$

$$L = 10.663 \log F_v + 34.64 \log \frac{1160 - 6V}{614 - 6V} - 60.44$$

Case 4 Condition No. 2; improved locomotive E:

Below 35 mph

$$F = \frac{44000}{1 + 0.00036V^2}$$

$$t = 0.045V + 0.0000054V^3$$

$$L = 0.000375V^2 + 0.0000000675V^4$$

Above 30 mph

$$F = 2.26V^2 - 804V + 55770$$

$$t = 12.07 \log \frac{1181.4 - 4.52V}{426.6 - 4.52V} - 5.19$$

$$L = 16.8 \log F_v + 35.79 \log \frac{1181.4 - 4.52V}{426.6 - 4.52V} - 95.47$$

Case 5 Condition No. 3; present locomotive A:

Below 35 mph

$$F = \frac{44000}{1 + 0.00052V^2}$$

$$t = 0.052V + 0.000009V^3$$

$$L = 0.000435V^2 + 0.000000112V^4$$

Above 30 mph

$$F = 4.883V^2 - 1142V + 59400$$

$$t = 13.89 \log \frac{1521.5 - 9.766V}{762.5 - 9.766V} - 4.02$$

$$L = 8.992 \log F_v + 27.09 \log \frac{1521.5 - 9.766V}{762.5 - 9.766V} - 51.04$$

Case 6 Condition No. 3; improved locomotive E:

Below 35 mph

$$F = \frac{44000}{1 + 0.000417V^2}$$

$$t = 0.052V + 0.0000072V^3$$

$$L = 0.000435V^2 + 0.00000009V^4$$

Above 30 mph

$$F = 0.5332V^2 - 607.7V + 49854$$

$$t = 10.28 \log \frac{1120.5 - 1.0664V}{94.9 - 1.0664V} - 11$$

$$L = 82.37 \log F_v + 97.62 \log \frac{1120.5 - 1.0664V}{94.9 - 1.0664V} - 491.57$$

Case 7 Condition No. 4; present locomotive A:

Below 35 mph

$$F = \frac{44000}{1 + 0.0004V^2}$$

$$t = 0.0305V + 0.00000407V^3$$

$$L = 0.000254V^2 + 0.000000051V^4$$

Above 30 mph

$$F = 2.52V^2 - 795.8V + 51860$$

$$t = 9.305 \log \frac{1128.2 - 5.04V}{463.4 - 5.04V} - 3.59$$

$$L = 10.23 \log F_v + 24.4 \log \frac{1128.2 - 5.04V}{463.4 - 5.04V} - 57.71$$

Case 8 Condition No. 5; present locomotive A:

Below 35 mph

$$F = \frac{44000}{1 + 0.000395V^2}$$

$$t = 0.0372V + 0.0000049V^3$$

$$L = 0.000323V^2 + 0.000000061V^4$$

Above 30 mph

$$F = 3V^2 - 870V + 54000$$

$$t = 11.42 \log \frac{1200 - 6V}{540 - 6V} - 3.94$$

$$L = 10.468 \log F_v + 27.60 \log \frac{1200 - 6V}{540 - 6V} - 59.09$$

Case 9 Condition No. 4; improved locomotive *E*:

Below 35 mph

$$F = \frac{44000}{1 + 0.00031V^2}$$

$$t = 0.0305V + 0.00000316V^3$$

$$L = 0.000254V^2 + 0.00000004V^4$$

Above 30 mph

$$F = 1.76V^2 - 684V + 53800$$

$$t = 10.36 \log \frac{982.5 - 3.52V}{385.5 - 3.52V} - 4.14$$

$$L = 14.65 \log F_v + 33.57 \log \frac{982.5 - 3.52V}{385.5 - 3.52V} - 82.89$$

Case 10 Condition No. 5; improved locomotive *E*:

Below 35 mph

$$F = \frac{44000}{1 + 0.000318V^2}$$

$$t = 0.0372V + 0.00000395V^3$$

$$L = 0.000323V^2 + 0.000000049V^4$$

Above 30 mph

$$F = 2.4375V^2 - 792V + 57274$$

$$t = 14.35 \log \frac{1054.5 - 4.875V}{529.5 - 4.875V} - 4.11$$

$$L = 12.88 \log F_v + 38.87 \log \frac{1054.5 - 4.875V}{529.5 - 4.875V} - 72.40$$

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Discussion

H. B. OATLEY.³ This paper presents, in easily understandable form, some of the essentials in the consideration of increased steam-locomotive power at the higher speeds which are today necessary. The thesis set forth is one in which the writer can heartily concur. There is only one point to which he would like to direct attention. It is the stress laid upon enlarged steam areas throughout the path of the steam from the dome to the exhaust-nozzle tip.

Chapelon, in his comprehensive analysis⁴ of the steam locomotive, the essential part of which has been so ably presented in English by Lawford H. Fry,⁵ has indicated the increased capacity resulting from careful attention to the question of more ample steam path. In the path of the steam as here considered, the fact must be kept in mind that all of the passages, with the exception of the superheater units, are of metal having a lower temperature than that of the steam carried. The superheater units, however, in part surrounded by gases at temperatures of upward of 1500 F, must be recognized as severely stressed parts. The steam velocity through these units is an important factor in preventing rapidly destructive conditions and it is essential that adequate steam velocity must be provided.

In the early days of the superheater, when materials less heat-resistant were available, steam velocities with the single-loop superheater were many times noticeably low; and the rapid deterioration of superheater units soon led to modified designs which provided higher steam velocities and better protection to the metal of the units. Today, with improved alloy steels, it is the practice to design for lower steam velocities than would otherwise be permissible, but it must be recognized that, even with these improved materials, too low steam velocities will result in unduly short life of this portion of the steam path. Like most important features in an engineering structure, there must be a compromise and all factors must be fairly evaluated. On one hand there is the desirability of the maximum output; on the other hand, the initial cost and maintenance, as well as the losses which may be incurred in repairs, must be considered.

L. K. SILLCOX.⁶ The author has presented, in intelligent and consecutive form, an example of the problem that is recurrent in both operating and mechanical departments of railways, i.e., the estimation of locomotive performance in terms of cars, tonnage, and speed. It is understood that the locomotives compared, designated *A* and *E*, are identical in principal dimensions and that the cylinder-horsepower performance of locomotive *E* has been achieved by securing a higher mean effective pressure in the cylinders by reducing pressure drop between superheater header and the cylinders, and by so modifying the valve events that an increase in negative work is not a function of short cutoffs at high speed.

The writer would expect that locomotive *E* indicates some hypothetical performance which locomotive *A* can never be re-

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⁴ "La Locomotive à Vapeur," by André Chapelon, published by J. B. Baillière et Fils, Paris, 1938.

⁵ "The Evolution of the Locomotive in France," by Lawford H. Fry, *Railway Mechanical Engineer*, vol. 112, 1938, pp. 473-475; vol. 113, 1939, pp. 1-5.

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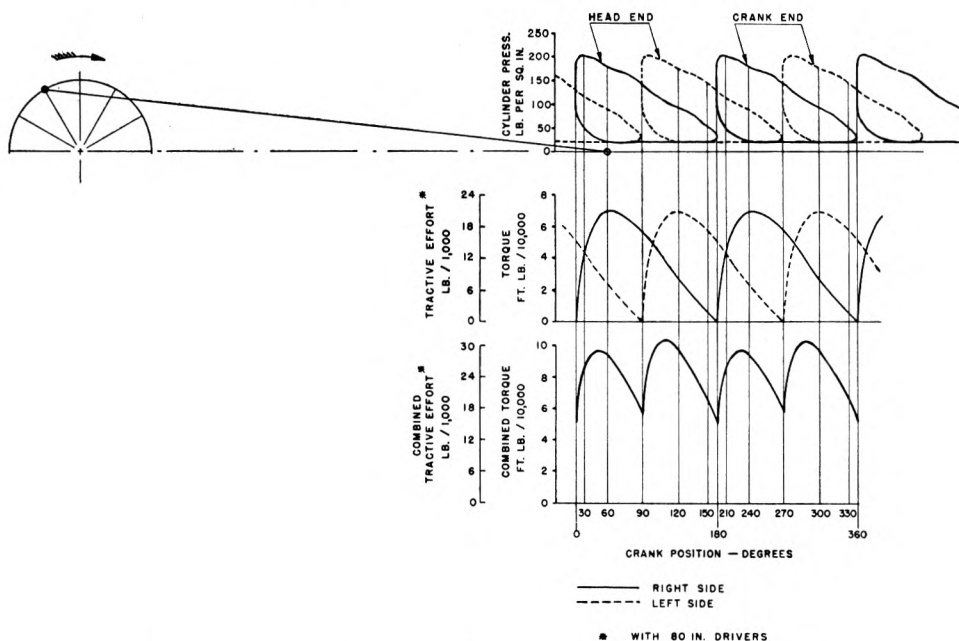


FIG. 12 COMPOSITE INDICATOR CARD FOR BOTH SIDES OF LOCOMOTIVE

constructed to produce economically. For instance, if the boiler will supply steam to deliver 4500 cylinder horsepower at 100 mph with a high degree of efficiency, its very size must be a burden to locomotive *A* which can do no better than a little more than 3400 hp at 60 mph. If maximum tractive effort is the same in either case, the same weight on drivers would serve either locomotive. The large boiler of locomotive *E* would demand more weight on idle axles.

Tractive effort expressed as

$$T = \frac{C^2PS}{D}$$

represents in simple form the average torque of a two-cylinder steam locomotive. Deduct total train resistance and it is a measure of accelerating capacity but is not an accurate and precise expression, because an equivalent mean effective pressure cannot be utilized to utmost advantage, bearing in mind the variation in instantaneous values of cylinder pressure during admission and expansion of steam, the effective lever arm of the couple which turns the main driving wheel on that side, and the manner in which the forces developed by two engines, operating 90 deg out of phase, combine. Increased mean effective pressure, however it may be obtained without proportionate increase in steam consumption or wider variation in extreme pressures, is desirable. The actual shape of the composite indicator card for both sides of the locomotive as it affects uniformity of torque also is important as disclosed by Fig. 12 of this discussion.

Since driving-wheel torque must be variable, there is a question as to the possible use which may be made of the peaks in accelerating force. At low speeds, surges are felt and at each surge some energy is wasted through movement of friction-draft-gear elements. This dissipation determines the number of cars through which the surge is discernible. If only the spring elements of the draft gears are affected a substantial part of the stored energy may be recovered. Maximum and minimum tractive efforts, developed from indicator cards, have been measured at 125 per cent and 76 per cent of the average values derived from the combined efforts of both engines at starting in

full gear and at 30 mph, 73 per cent cut off. Obviously, no increase in mean effective pressure, secured by increasing instantaneous high pressures at crank positions which now are associated with the highest maximum torques, would be effective. At the same time, minimum torque occurs when the crankpins are at or near the center and quarter positions. At these points, any reduction in negative work, represented by compression and preadmission, is highly beneficial in the direction of useful and uniform tractive effort. Thus, a valve gear, which will provide independent timing of separate events, will be of considerable value in smoothing out the torque curve, quite independently of its effect upon mean effective pressure.

AUTHOR'S CLOSURE

Mr. Oatley has rightly called attention to the necessity for proper balance in all things, particularly in the steam locomotive. It is fully recognized that freer steam and gas passages should not be realized at the expense of either superheat or tube maintenance. It is rather the author's contention that our enthusiasm for evaporative surface and superheat has combined with increasing demands on the capacity of the locomotive as a whole in such a way as to swing the balance away from adequate areas. Steam and gas passages which were sufficient twenty or thirty years ago are too restrictive today, and our conceptions of proper balance must therefore submit to some overhauling.

Mr. Sillcox has pertinently pointed out that increased mean effective pressure without increase in boiler pressure can be brought about by eliminating negative work. Bearing in mind that the greatest improvement is realizable at the higher speeds, a large increase also comes from improved admission-valve action and later exhaust opening, which combine to increase the positive work available from the same amount of steam.

When this improvement in the cylinder cycle is supplemented by a higher average steam-chest pressure and a lower average exhaust back pressure brought about by refinements in other details of the machine, locomotive *E* is no longer a hypothetical case but becomes an attainable reality. The same boiler will serve because the steam delivered to the cylinders is the same in both cases.

Discharge Coefficients of Long-Radius Flow Nozzles When Used With Pipe-Wall Pressure Taps

By H. S. BEAN,¹ S. R. BEITLER,² AND R. E. SPREngle³

For the last six years, the Special Research Committee on Fluid Meters has been conducting a research on flow nozzles. In previous papers relating to the program, the Committee's plans for carrying out the research were outlined (1),⁴ and some of the results obtained have been presented (2, 3, 4). In the present paper, the results of determinations of coefficients of long-radius nozzles when used with pipe-wall taps⁵ are given, as based on a combination of separate analyses by the three authors. Later papers will extend the results to other conditions than those considered in this paper.

INTRODUCTION

AS WAS pointed out in one of the earlier papers (1), there is no one laboratory with facilities for making the required tests on flow nozzles over the entire range of conditions which it was desired to cover. Furthermore, by distributing the tests among several laboratories, more or less overlapping would occur, which would tend to furnish information on the agreement between the results of tests by the different laboratories on the same nozzle; in some cases using the same sections of piping. In addition, such data would provide a basis for estimating the tolerance to be assigned to the coefficient values for use in commercial metering of fluids. Since it is impossible to determine, from the smoothed results as here given, the portions contributed by the different laboratories, it is appropriate to list them and to indicate the range of conditions covered by their tests, so that due recognition may be given to their several contributions. This is done in Table 1.

METHOD OF ANALYSIS

For the purpose of the present paper, the essential result from each test is the relation between the discharge coefficient and the Reynolds number. The discharge coefficient is defined (5) by

$$w = 0.525 \frac{CD_2^2}{\sqrt{1-\beta^4}} \sqrt{(p_1 - p_2)\rho_1} \dots \dots \dots [1]$$

in which w = actual rate of flow, lb per sec
 C = coefficient of discharge
 β = diameter ratio, D_2/D_1

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⁴ Numbers in parentheses refer to the Bibliography at the end of the paper.

⁵ As here used, the term "pipe-wall taps" applies to pressure taps connected to the pipe wall with the inlet-pressure tap located 1 pipe diam ahead of the nozzle-inlet face, and the outlet pressure tap about 1/2 pipe diam following the nozzle-inlet face, but in no case beyond the outlet end of the nozzle.

Contributed by the Special Research Committee on Fluid Meters and presented at the Annual Meeting, New York, N. Y., December 2-6, 1940, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

D_1 = inside diameter of approach section of pipe, in.
 D_2 = diameter of nozzle throat, in.
 p_1 = absolute static pressure on inlet side of nozzle, psi
 p_2 = absolute static pressure on outlet side of nozzle, psi
 ρ_1 = density of fluid based on inlet pressure and temperature, lb per cu ft

The Reynolds number R_d , applying to the nozzle throat diameter, is given by

$$R_d = \frac{V_2 D_2 \rho_1}{12 \mu_1} \quad \text{or} \quad R_d = \frac{48 w}{\pi D_2 \mu_1} \dots \dots \dots [2]$$

while R_D , applying to the pipe diameter, is given by

$$R_D = \frac{V_1 D_1 \rho_1}{12 \mu_1} \quad \text{or} \quad R_D = \frac{48 w}{\pi D_1 \mu_1} \dots \dots \dots [3]$$

V_1 = average fluid velocity in the pipe, fps

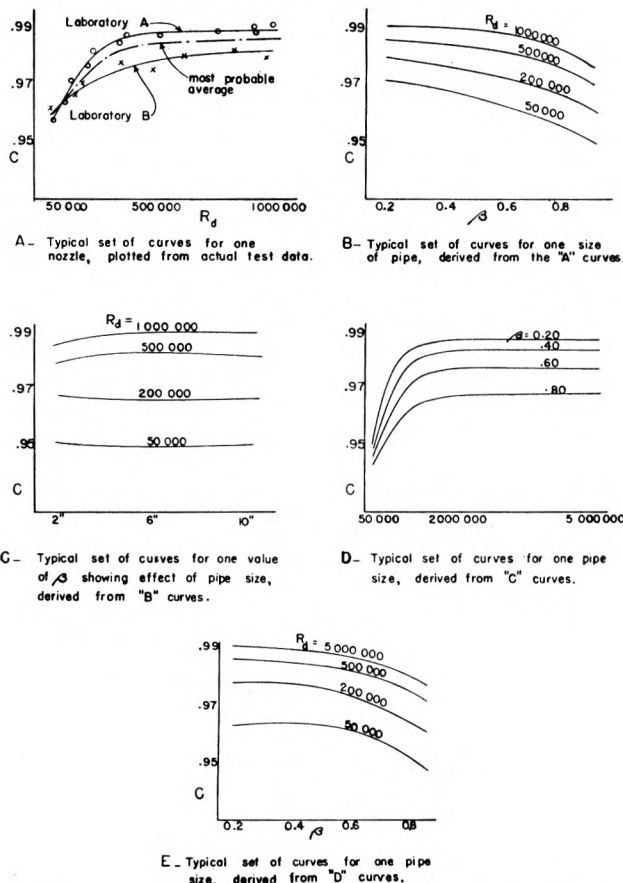


FIG. 1 SCHEMATIC ILLUSTRATION OF GRAPHICAL METHOD USED IN ANALYZING FLOW-NOZZLE TEST DATA

V_2 = average fluid velocity in nozzle throat, fps

μ_1 = absolute viscosity of fluid at inlet pressure and temperature, lb per ft per sec

From Equations [2] and [3] it will be seen that

$$R_D = \beta R_d \dots \dots \dots [4]$$

a relation which will be referred to again.

The first step in the analysis of the results was to plot all the values of C determined for any one nozzle, against the corresponding values of the Reynolds number. A smooth curve was then drawn to represent the locus of the test points as closely as possible, as illustrated in *A*, Fig. 1. When more than one laboratory calibrated a given nozzle, the results from each laboratory were plotted separately, or with distinguishing marks. Smooth curves were then drawn to represent the results from each laboratory. Finally, a single curve was drawn to represent what the analyst believed to be the most probable average of all the data taken on that one nozzle. The result of this step was that an average curve was obtained for each nozzle tested.

Next, from the average curves for all nozzles tested in any one size of pipe, values of C were read off corresponding to some even value of the Reynolds number and these were plotted against the diameter ratio. Smooth curves were then drawn to connect the points for like values of the Reynolds number as in *B*, Fig. 1. This process was repeated for each different size of pipe in which nozzles were tested. As a matter of detail, it may be mentioned that it was usually necessary to cross-plot back and forth several times before sufficiently smooth curves were obtained in both the *A* and *B* plots.

For an even value of the diameter ratio, values of C at a given Reynolds, number were read from each group of *B* curves and plotted against pipe diameter (or the reciprocal of the pipe diameter). In this manner groups of curves were obtained for each selected value of the diameter ratio, as shown in *C*, Fig. 1. These curves show the change in the coefficient due to pipe size. While this effect was small and rather irregular, it was deemed desirable to give it some consideration in this analysis.

From these sets of curves, giving the effects of pipe size, values of the discharge coefficient were read for a single pipe size and plotted against Reynolds' number. Smooth curves were readily drawn through the points for the same diameter ratio, and thus for each pipe size was formed a family of curves giving coefficient against Reynolds' number for even diameter ratios, as illustrated in *D*, Fig. 1. Each of these families of curves was extrapolated to a Reynolds number R_d of 5,000,000, in order to meet the conditions in many present-day plants, particularly power plants where very high steam temperatures and velocities are used.

It will be noted that the curves in group *D* are similar to those of group *A*, but are much smoother and more evenly spaced. The analysis could have been ended here, and this set of *D* curves used to present the final results. However, it was the authors' belief that most users of flow nozzles would find it more convenient to have available curves in which the coefficient is plotted against diameter ratio. Accordingly, the final step was to make cross-plots from the *D* curves, thus giving curves of C for constant Reynolds' numbers against diameter ratio, as shown in *E*, Fig. 1, a separate set of curves being made for each pipe size.

As already mentioned, each of the authors made independent analyses of the data, following the general procedure outlined.

TABLE I

Laboratories in Which Flow Nozzle Tests Were Made and Range of Conditions Covered

Laboratories in Which Flow Nozzle Tests Were Made and Range of Conditions Covered											
Laboratory	Size of Pipe inches	Number Nozzles Tested	Range of Diam- eter Ratios		Fluid	Range of Reynolds Numbers				Reference Meas- urement by	Tests Made or Supervised by
						R_D		R_d			
						min	max	min	max		
Bailey Meter Co	3	13	.16	to .80	water	6 600	870 000	22 000	1 090 000	volumetric tank	R. E. Sprengle
Cornell University, Hy- draulic Laboratory	8	1	.46		water	18 000	294 000	39 700	633 000	volumetric tank	Prof. E. W. Schoder, A. N. Vanderlip
	24	3	.24	.56	"	23 600	1 012 000	97 600	2 357 000	" "	
Cornell University, Sibley School of M.E.	3	4	.25	.60	water	3 000	400 000	12 700	670 000	weigh tank	Prof. s F. G. Switzer & W. C. Andrae
	4	8	.35	.85	"	9 000	596 000	25 400	781 000	" "	
General Electric Co. Tur- bine Research Dept.	2	8	.50	.84	steam	117 400	1 377 000	243 000	1 690 000	weighed condensate	B. O. Buckland, C. J. Walker
Ingersoll-Rand Co. Test- ing Dept.	4	2	.25	.50	air	6 000	59 500	24 000	119 000	impact tube	R. W. Johnson, R. E. Hunn
	16	9	.24	.36	"	51 000	386 000	200 000	1 700 000	traverse	
Massachusetts Institute of Technology, Dept. of ME	2	5	.125	.50	steam	4 400	128 600	34 900	257 200	weighed condensate	Prof. J. H. Keenan, W. J. Lindsey
National Bureau of Standards	2	20	.125	.84	water	155	620 000	1 290	768 000	weigh tank	H. S. Bean, F. C. Morey, H. G. Wagner, H. P. Bean, B. O'Connor
	3	17	.16	.86	"	426	628 000	2 900	734 000	" "	
	4	16	.15	.85	"	500	403 000	3 260	480 600	" "	
	8	10	.37	.86	"	10 700	783 000	28 400	986 000	volumetric tank	
Ohio State University, Dept. of M. E.	3	13	.16	.86	water	1 300	755 000	4 200	1 068 000	weigh tank	Prof. S. R. Beitter
	3 XH	6	.16	.86	steam	43 000	969 000	160 000	1 500 000	weighed condensate	
	6 XH	5	.54	.79	water	45 300	1 367 000	84 000	1 737 000	weigh tank	
	8 XH	6	.69	.82	"	57 700	1 952 000	82 000	2 672 000	volumetric tank	
	8	10	.37	.86	"	38 000	1 756 000	102 000	2 064 000	" "	
	10 XH	3	.82		"	226 000	1 626 000	277 000	2 110 000	" "	
	12	1	.64		"	112 000	1 418 000	175 000	2 205 000	" "	
University of California Dept. of M. E.	3	3	.4	.8	water	26 000	692 000	65 000	865 000	volumetric tank	Prof. s M. P. O'Brien & R. G. Folsom
	4	8	.35	.85	"	78 700	754 000	127 000	868 000	" "	
	8	9	.37	.56	"	62 000	549 000	160 000	984 700	" "	
University of Oklahoma, School of M. E.	2	22	.125	.84	oil	60	60 100	493	61 200	weigh tank	Dean W. H. Carson, Prof. E. E. Ambrosius
	3	19	.16	.86	"	48	49 500	290	69 700	" "	
	4	14	.15	.85	"	6	37 800	40	50 700	" "	
University of Pennsylvania, Dept. of C. E.	4	5	.37	.85	water	4 000	852 000	10 800	990 000	weigh tank	Prof. W. S. Pardoe
	6	5	.25	.58	"	4 000	590 000	12 000	1 020 000	" "	
	8	2	.46	.83	"	7 600	1 650 000	15 000	2 000 000	" "	

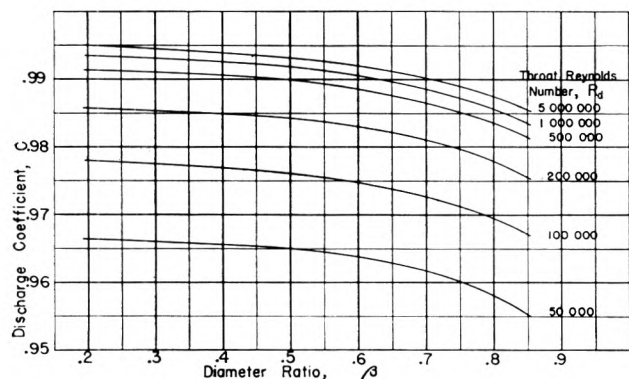


FIG. 2 DISCHARGE COEFFICIENTS OF LONG-RADIUS NOZZLES IN 2-IN. PIPE
(When inlet-pressure connection is 1 pipe diam preceding and outlet-pressure connection is $1/2$ pipe diam following plane of nozzle inlet.)

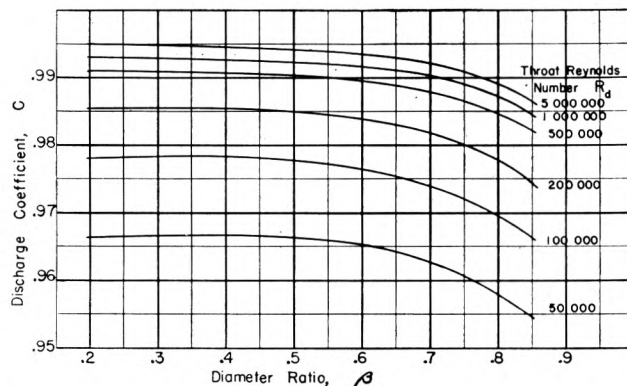


FIG. 5 DISCHARGE COEFFICIENTS OF LONG-RADIUS NOZZLES IN 6-IN. PIPE
(Refer to note Fig. 2.)

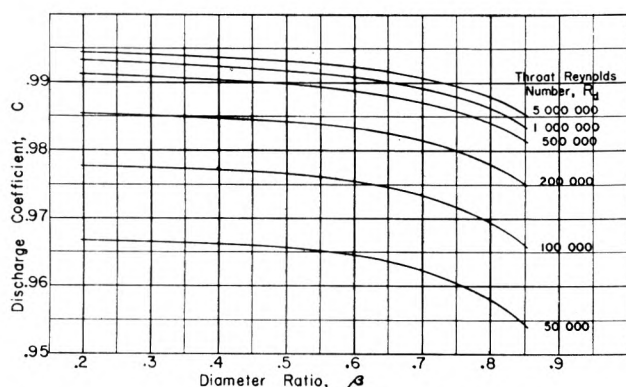


FIG. 3 DISCHARGE COEFFICIENTS OF LONG-RADIUS NOZZLES IN 3-IN. PIPE
(Refer to note Fig. 2.)

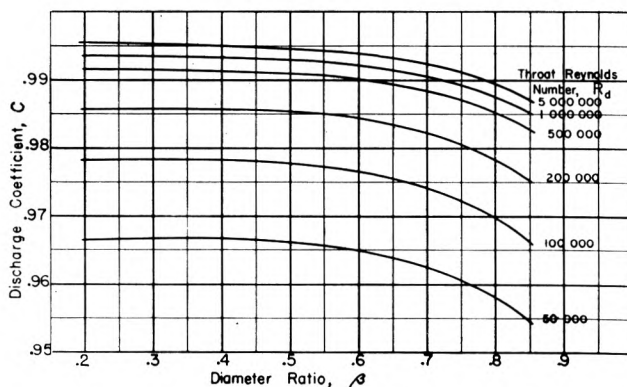


FIG. 6 DISCHARGE COEFFICIENTS OF LONG-RADIUS NOZZLES IN 8-IN. PIPE
(Refer to note Fig. 2.)

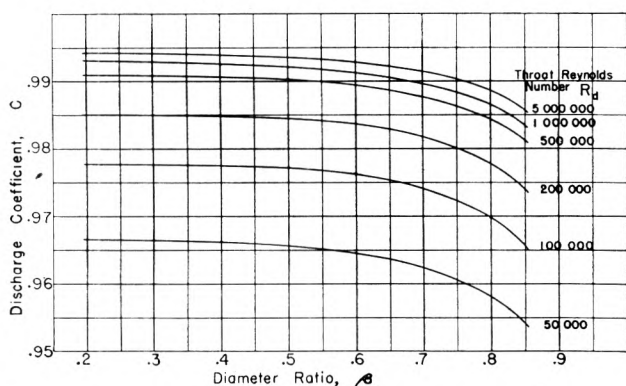


FIG. 4 DISCHARGE COEFFICIENTS OF LONG-RADIUS NOZZLES IN 4-IN. PIPE
(Refer to note Fig. 2.)

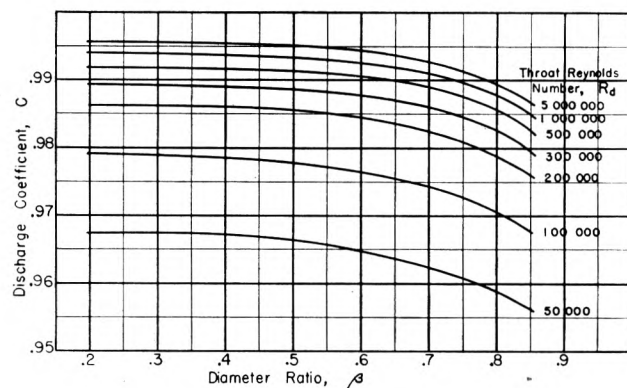


FIG. 7 DISCHARGE COEFFICIENTS OF LONG-RADIUS NOZZLES IN 10-IN. PIPE
(Refer to note Fig. 2.)

There was, however, more or less variation in the details of performing the several steps. For example, in plotting the original test data in the first step, either R_D or R_d may be used as the abscissa coordinate. Also, it was sometimes more convenient to use the logarithm of the Reynolds number, or to plot on semi-logarithmic coordinate paper. Whichever Reynolds' number was used in the first step was used ordinarily throughout the succeeding steps. However, for the final comparison, only one was used, in this case R_d . This means that, when R_D was used in the first four steps, it was necessary to apply Equation [4] to the curves in D , Fig. 1, before cross-plotting to obtain the final

set of E curves. This shifted each curve in D to the right, but by different amounts, the lower the diameter ratio the greater the amount of shift. It is evident that this alters the shape of the curves which are obtained in the final step by cross-plotting the D group of curves.

Furthermore, each author used his own estimation of the weight to be placed on the results from the different laboratories, particularly when two or more laboratories calibrated the same nozzle. These three individual sets of curves were then compared, line by line and the arithmetical mean taken as the basis for the curves herein presented.

COEFFICIENTS FOR LONG-RADIUS FLOW NOZZLES, TOLERANCES, AND RANGE OF APPLICATION

The average curves obtained from the three analyses are shown in Figs. 2, 3, 4, 5, 6, and 7, applying to flow nozzles in 2-, 3-, 4-, 6-, 8-, and 10-in. pipes, respectively. These curves constitute the Committee's recommendation on the values of discharge coefficients for the long-radius or elliptical type of flow nozzle when the inlet-pressure connection is located 1 pipe diam preceding the inlet face of the nozzle, and the outlet-pressure connection is $1\frac{1}{2}$ pipe diam following the inlet face of the nozzle.

The average difference between the results reported here and any one of the three individual analyses is about ± 0.2 per cent, while the maximum difference is about 0.5 per cent. However, between the results reported by different laboratories on tests of the same nozzle, the differences were as much as 1 per cent to 1.5 per cent, even when the same sections of pipe had been used. As a rule, the differences were larger at the low Reynolds numbers than at the high. Therefore, the authors suggest that, in the use of these nozzle coefficients, a tolerance (i.e., the probable range uncertainty) of ± 0.75 per cent be allowed at Reynolds' numbers R_d of 500,000 and over, with 3-in. pipe and larger. At lower values of R_d and with 2-in. pipe, the tolerance should be ± 1 per cent.

While the coefficient curves for all six pipe sizes have been extended up to a diameter ratio of 0.85, it is recommended that the use of diameter ratios in excess of 0.8 should be avoided wherever possible. The reasons for this recommendation are (a) the test data for diameter ratios over 0.8 were less extensive and more irregular than for the lower values of the ratio; (b) the slope of the curves is increasing rapidly in the 0.8 to 0.85 region, and the errors due to any uncertainty in the calculation of the diameter ratio or to reading the coefficient value from the curves will be greater than at the lower values of β .

A careful comparison of the curves for use with the different pipe sizes will show there is very little change in the value of the coefficient with pipe size, particularly with the larger pipes. Any further change in coefficient values with pipe size above a 10-in. pipe will probably be very slight. Therefore, it is sug-

gested that for nozzles in pipes larger than 10 in. the coefficient curves for 10-in. pipe may be used without introducing any appreciable error.

ACKNOWLEDGMENTS

The conduct of any research project, and particularly so extensive a program as this has been, requires ample financial support. Too often the efforts to provide this support pass unnoticed, and that such may not occur in this case, we are pleased to record here that the carrying on of this program was made possible by the efforts of E. C. M. Stahl as chairman of the fund-raising subcommittee. Assisting him on this subcommittee were R. K. Blanchard, W. A. Carter, Paul Diserens, and L. K. Spink. For their repeated and generous response to the requests from this subcommittee particular acknowledgment is due The Engineering Foundation and the former Utilities Coordinated Research, Inc. The names of all the contributors of both funds and materials (other than laboratory testing facilities) are given in Table 2.

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Discussion

W. A. CARTER.⁶ In connection with the graph showing the large spread of discharge coefficient of a certain flow-nozzle installation plotted against the Reynolds number, which appears in the authors' closure to this paper, it is noted that the installation involved corner taps.

The writer believes the authors will agree that such installations are much more susceptible to erratic performance than are those having the pressure taps located 1 pipe diam upstream and $1\frac{1}{2}$ pipe diam downstream from the nozzle inlet, which latter is a type of installation with which the paper is concerned.

Corner taps are known to be dependent upon the width of the pressure-slot opening if the I.S.A. nozzle design is employed. A small variation in the thickness of the gaskets may seriously affect the nozzle coefficient.

W. W. JOHNSON.⁷ Inasmuch as the data given in this paper supply authoritative values of nozzle-discharge coefficients over a wide region, where heretofore only meager information has been published, the paper will be greatly appreciated by engineers having flow problems to handle.

In order to clarify certain questions which have arisen will the authors supply answers to the following:

Are the coefficients given in the paper for low-ratio or high-ratio nozzles, as defined in "Instruments and Apparatus," part 5, chapter 4,⁸ or both?

What rules were followed in regard to wall thickness of the test

TABLE 2 CONTRIBUTORS TO THE FLOW-NOZZLE RESEARCH PROGRAM

American Gas and Electric Company
Anaconda Copper Mining Company
Bailey Meter Company
Bethlehem Steel Company
Burlington Lines
Central Illinois Light Company
Central Railroad of New Jersey
Chicago, Milwaukee, St. Paul and Pacific Railroad Company
Connecticut Light and Power Company
Consolidated Edison Company of New York, Inc.
De Laval Steam Turbine Company
Detroit Edison Company
Duquesne Light Company
Ebasco Services, Incorporated
Economy Pumping Machinery Company
The Engineering Foundation
Foxboro Company
Gulf Oil Corporation of Pennsylvania
Gulf Research and Development Corporation
Humble Oil and Refining Company
M. W. Kellogg Company
New England Power Association
New England Power Service Company
Norfolk and Western Railway Company
Pacific Pump Works
Philadelphia Electric Company
Potomac Edison Company
Public Service Electric and Gas Company
Soco-Vacuum Oil Company
Standard Brands, Incorporated
Standard Development, Incorporated
Sun Oil Company
United States Steel Corporation
United States Works Progress Administration
Washington Gas Light Company
West Penn Power Company
Union Electric Light and Power Company
Utilities Coordinated Research, Incorporated
Westinghouse Electric and Manufacturing Company
Worthington Pump and Machinery Corporation
Youngstown Sheet and Tube Company

⁶ Technical Engineer, Power Plants, The Detroit Edison Company, Detroit, Mich. Mem. A.S.M.E.

⁷ Research Engineer, Turbine Engineering Department, General Electric Company River Works, Lynn, Mass. Mem. A.S.M.E.

⁸ Information on Instruments and Apparatus, Part 5, Chapter 4, on Flow Measurement by Means of Standardized Nozzles and Orifice Plates, published by The American Society of Mechanical Engineers, 29 W. 39th St., New York, N. Y.

nozzles, and what would be the effect on the coefficients if relatively heavy walled nozzles were used?

Why have the coefficients not been given for throat Reynolds' numbers less than 50,000 and what, in general, was the result of the tests at the lower range of Reynolds' number?

W. V. KING.⁹ Two high-ratio flow nozzles for unit No. 7 at Waterside Station No. 2 of the Consolidated Edison Company have been calibrated at Ohio State University by S. R. Beitler. The results of these calibration tests present an interesting discussion to this paper because they tend to substantiate the opinion of the authors that it is difficult to predict the coefficient of flow nozzles with diameter ratios above 0.8 to any reasonable degree of accuracy.

Both of these nozzles were designed for 700,000 lb steam flow per hr at 1300 psi pressure and 925 F temperature. To limit the differential head to 212 in. of water at rated steam flow, it was necessary to use flow nozzles with a diameter ratio of 0.8366.

The Reynolds number on these nozzles, corresponding to rated

⁹ Assistant Engineer, Consolidated Edison Company, Inc., New York, N. Y. Mem. A.S.M.E.

steam flow, is 10,000,000, equivalent to nearly 3 times the maximum Reynolds number obtained on test with water. Therefore it was necessary to extrapolate the calibration tests by a method which appears to give the most accurate determination of coefficient at design conditions. This basis of extrapolation is the relation that the logarithm of the actual water flow plotted against the logarithm of differential head is a straight line. The method of extrapolation is explained as follows:

1 All test points were corrected to a constant temperature; the differential head by a factor equivalent to the ratio of water densities, and the actual water flow by a factor equivalent to the square root of the water-density ratio. To minimize the magnitude of these corrections, the variation of water temperatures during test was confined to 5 deg for nozzle No. 59535 and to 3 deg for nozzle No. 59534.

2 The equation of a straight line expressing the relation between the logarithms of actual flow and differential head is $\log Q = A + B \log h$, where A is the intercept and B is the slope of that line. The equation of the best line through the test points can be calculated most accurately by the method of least squares, applying the relations

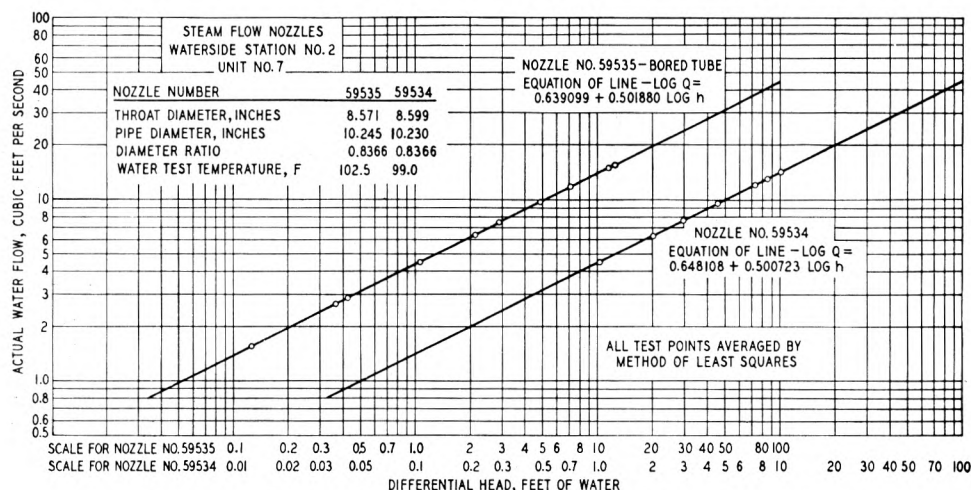


FIG. 8 LOGARITHMIC RELATION BETWEEN DIFFERENTIAL HEAD AND ACTUAL WATER FLOW

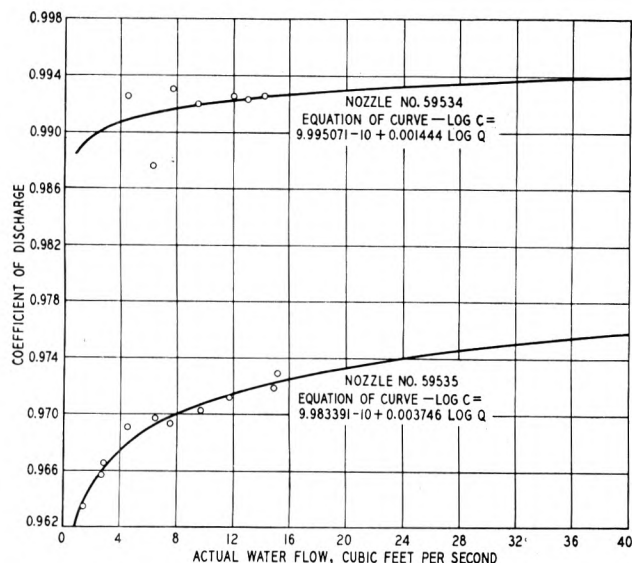


FIG. 9 COEFFICIENT OF DISCHARGE IN RELATION TO ACTUAL WATER FLOW

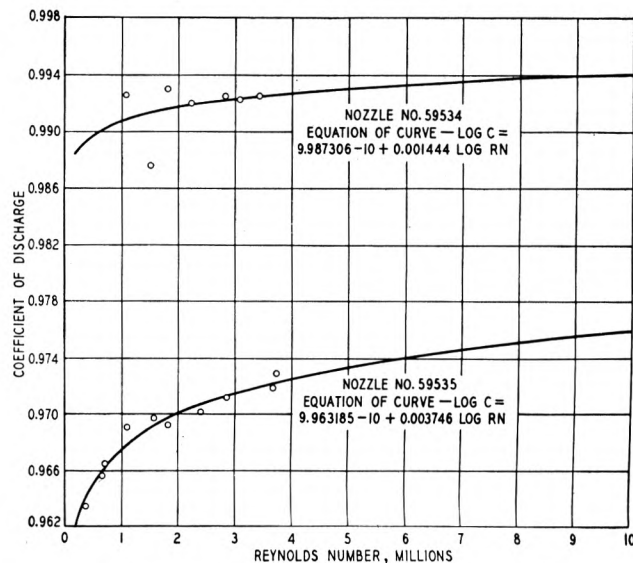


FIG. 10 COEFFICIENT OF DISCHARGE IN RELATION TO REYNOLDS' NUMBER

$$\Sigma B \log h + NA = \Sigma \log Q$$

$$\Sigma A \log h + \Sigma B (\log h)^2 = \Sigma \log Q \log h$$

where N is the number of test points.

3 Since the coefficient of discharge is equivalent to the actual water flow divided by the theoretical flow ($Q = K\sqrt{h}$), it is a simple mathematical procedure to determine the logarithm of the coefficient in terms of the logarithms of differential head, actual water flow, and Reynolds' number which are all straight lines.

The final calibration curves on these two nozzles are presented, first, to indicate the apparent accuracy of extrapolation and then to show the difference in discharge coefficient between two nozzles with the same diameter ratio of 0.8366.

The coefficient curves for nozzle No. 59535 installed in a bored tube do not vary from any test point by more than 0.1 per cent and the curve is the best average of the test points. Unfortunately, the coefficient curves for nozzle No. 59534, installed in an unbored tube, pass through four test points, fall under two test points by approximately 0.15 per cent, and lie above one test point by approximately 0.35 per cent. With the exception of one test point, the calibration-test accuracy falls within 0.15 per cent of the faired curves.

A comparison of these coefficient curves indicates that nozzle No. 59534 has a coefficient approximately 2 per cent greater than nozzle No. 59535, and also that the coefficient for nozzle No. 59534 is relatively flat compared to the coefficient for nozzle No. 59535.

Experience with high-ratio flow nozzles has prompted the following comments on paper:

1 The plotting of a smooth curve, representing the locus of test points, by an arithmetical average may be sufficiently accurate when the spread of points from the faired curve is not excessive and when the test covers the entire range of Reynolds' numbers. Whenever an extensive extrapolation is required, it is preferable to use the method of least squares in averaging the test points.

2 The straight-line relation between logarithms of flow and head discloses that a plot of the logarithm of coefficient against logarithms of differential head, actual water flow, and Reynolds' number also have a straight-line relation. Therefore, any direct plot of coefficient against Reynolds' number might introduce inaccuracy, provided the coefficient curve has a steep slope and test points cover a great range of Reynolds' number.

The authors are to be commended on presenting these extensive test data on flow nozzles in such a concise manner. Due to the tremendous amount of work necessary in preparing this paper, the methods used by the authors in extrapolating and averaging

coefficient curves appear to be the most suitable that could be applied under the circumstances. This discussion has been presented to introduce additional data on the higher-diameter-ratio nozzles and to show what appears to be a more desirable method of extrapolating test data on coefficient of discharge to procure the greatest accuracy possible.

W. S. PARDOE.¹⁰ From an inspection of the graph giving the coefficient curves of high-ratio flow nozzles, with pipe taps, which appears in the closure to this paper, it is suggested that the authors might do well to consider the use of throat taps for high-ratio nozzles.

Fig. 11 of this discussion shows the test of an 11.064 × 9.346-in. flow nozzle with throat taps. The coefficient curve is quite normal and is flat from Reynolds' number 200,000 up.

AUTHORS' CLOSURE

Mr. Carter refers to Fig. 12 of this closure which shows the results of tests on large-diameter-ratio nozzles when corner taps were used. The purpose of this illustration is to emphasize the difficulty of correlating the results when there is so much scatter-

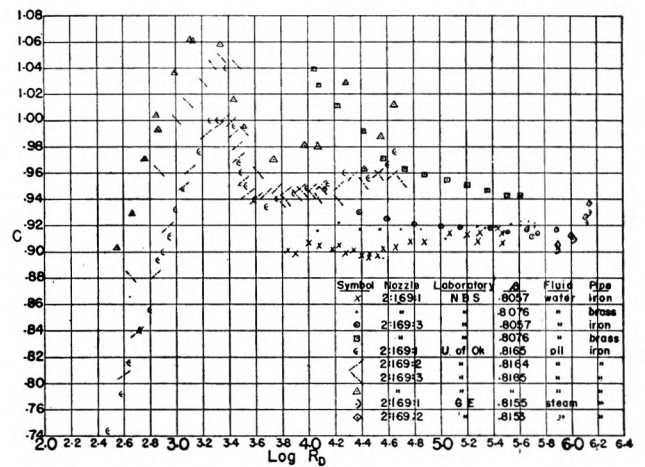


FIG. 12 RESULTS OF TESTS WITH CORNER TAPS ON THREE NOZZLES OF NEARLY THE SAME THROAT DIAMETER

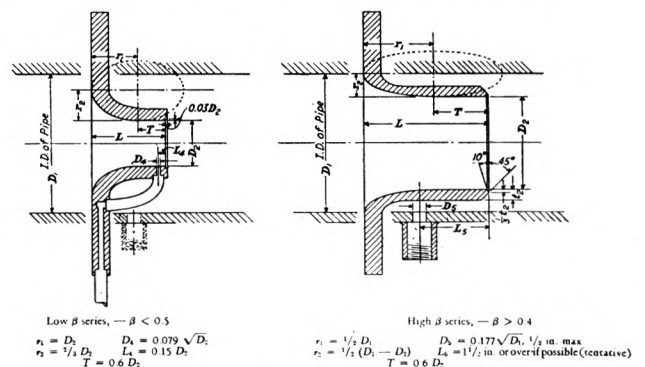


FIG. 13 PROPORTIONS OF LONG RADIUS FLOW NOZZLES ADOPTED BY THE SUBCOMMITTEE ON FLOW-NOZZLE RESEARCH FOR USE IN ITS RESEARCH PROGRAM

(The use of a pipe-wall tap instead of a nozzle-throat tap in the low β series is optional.)

ing. It is for the reason that there is more scattering with corner taps, as mentioned by Mr. Carter, that this particular plot is re-

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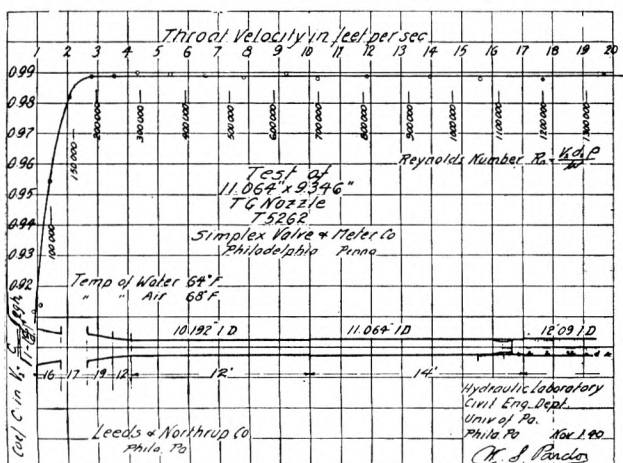


FIG. 11 TEST OF 11.064 × 9.346-IN. TG NOZZLE (Simplex Valve & Meter Company, Philadelphia, Pa.)

produced. It may be added that the location of the pressure taps, as between corner taps and pipe-wall taps, is more important than the nozzle-entrance forms, as represented by I.S.A. and long radius.

In reply to Dr. Johnson's questions, as shown by the curves in Figs. 2 to 7, inclusive, of the paper, the coefficients given cover diameter ratios all the way from 0.2 to over 0.8, so that they apply to both low-ratio and high-ratio nozzles, as these terms were originally defined by the committee. The meanings of the terms "low-ratio" and "high-ratio" are clearly defined and illustrated in Fig. 13¹¹ of this closure. It should be noted that the nozzle proportions given in Fig. 20 of Dr. Johnson's reference⁸ do not correspond exactly to those given in Fig. 12 just referred to. The most important difference is in the length of the parallel portion of the throat. Except for a few nozzles loaned to the committee, the length of the throat section of all nozzles did not exceed the $0.6D_2$ shown in Fig. 13 of this closure. Of course, with the I.S.A. nozzles that were used, this throat section is even shorter.

It has been known for some time¹² that a long throat section will result in lowering the value of the discharge coefficients slightly. This will be particularly true as the throat diameter is decreased. Therefore, it cannot be expected that the coefficient values given in the present paper will apply exactly to small-diameter nozzles if made with as long a throat section as is called for by Dr. Johnson's reference.⁸ It is unfortunate that such a discrepancy should exist between values given in papers by two committees of the Society. The authors do not know the source of Fig. 20 of that report.⁸ However, as one of the authors is a member of the subcommittee which prepared the former report, he accepts his share of the responsibility for the difference between the two figures.

As to the wall thickness of the nozzle throat there was no general rule. For the larger-diameter-ratio nozzles, there was the requirement that there be some clearance (over $1/16$ in. diam) between the outside surface of the nozzle throat and the inner surface of the pipe. The purpose of this clearance is to provide fluid passage to the outlet-pressure tap when this is "under" the nozzle throat. This made it necessary to have the throat-wall thickness about $1/8$ in. for some of the larger nozzles for 4-in. and smaller pipe. For most of the nozzles the throat-wall thickness ranged from about $1/16$ in. for 2-in. pipe to $5/8$ in. for 8- and 16-in. pipe. So far as the authors know, the principal advantage of a thick wall is to diminish the possibility of the nozzle being damaged or deformed in handling.

The authors believe that the range of Reynolds' numbers covered in the present paper probably meets 75 per cent or more of the cases in commercial use. Moreover, there is much less scattering of the test data in the higher Reynolds' number range

so that it was easier to obtain agreement over that region. Therefore, it seemed advisable to present this much of the results so users could have the benefit of it. As shown by Table 1 of the paper, some data were obtained for throat Reynolds' numbers of less than 100 and these results will be presented in a later paper. However, the spread of coefficients at low Reynolds' numbers is much greater than for the region here considered, so that they will be subject to a much larger tolerance than given for these results.

As to Professor Pardoe's suggestion, some eight to twelve nozzles provided with throat taps were used in the research program, although none was much if any over 0.5-diam ratio. Two of the nozzles were equipped with four pressure holes at 90 deg. With both of these, the results were more or less different with each pressure hole. While such a limited number of tests is insufficient as a basis for a final conclusion, it does strengthen the authors' belief that placing a pressure hole in a nozzle throat, where the fluid velocity past the hole is the highest, adds to the difficulties of obtaining reproduceable conditions. Moreover, except where the nozzle is of the solid-block type, the use of a throat tap adds considerably to the cost of construction.

Mr. King has shown a method which may be followed where it becomes necessary to extrapolate far beyond the range of the test data. The authors agree that, if practicable, for such cases, it is always better to use a method of plotting which results in straight lines.

However, it should be pointed out that there are but scant published data to support the assumption that the actual flow is:

$Q = Kh^n$, where n is a value different from the theoretical value of 0.5, which is the assumption upon which this straight line is drawn. It is also apparent that, if the curve is to be used for extrapolation, the head scale is being extrapolated 9 times its highest reading if the quantity and Reynolds' number curves are being extrapolated to 3 times their value.

These facts seem to indicate that Mr. King's suggested method requires more proof as to its accuracy before it can be accepted as the best method of extrapolation of these data.

Referring to the difference in coefficients between Mr. King's nozzles No. 59534 and No. 59535, the authors believe this illustrates the desirability, not only of keeping the ratio of throat diameter to pipe diameter at or below 80 per cent whenever possible, but also of boring the pipe so as to make the pipe surface concentric with the nozzle throat and as smooth and free from local irregularities as possible. Naturally, the effect of such surface roughness and eccentricity is greatly minimized with nozzles of small diameter ratio, but quite likely to be magnified as the diameter ratio increases.

In comparing the shape of the coefficient curves of these two nozzles in Fig. 10 of Mr. King's discussion, it must be remembered that twice as much data were taken on the bored-pipe nozzle No. 59535 as with the unbored-pipe nozzle No. 59534; and that, had a slight amount of data been taken with nozzle No. 59534, the shape of its coefficient curve would likely have been much the same as the curve for the bored-pipe nozzle.

¹¹ Reproduced from paper, "Research on Flow Nozzles," by H. S. Bean, *Mechanical Engineering*, vol. 59, 1937, Fig. 1, p. 501.

¹² "Measurement of Flow of Air and Gas With Nozzles," by S. A. Moss, *Trans. A.S.M.E.*, vol. 50, 1928, paper APM-3, pp. 1-10; discussion by H. S. Bean, pp. 13-15.

Remarks on the Analogy Between Heat Transfer and Momentum Transfer

BY L. M. K. BOELTER,¹ R. C. MARTINELLI,² AND FINN JONASSEN²

This paper presents an extension of Von Kármán's analysis of heat transfer to fluids in closed conduits, based on the analogy between heat transfer and momentum transfer. For a particular ideal system, an expression for the temperature distribution in a fluid in turbulent motion being heated or cooled inside of a circular pipe is derived and a relation is obtained between Nusselt's modulus and the pipe-friction factor for "isothermal" heat transfer. This equation is extended to apply in cases in which the physical properties of the fluid vary across the section of the pipe. The apparent variation of the dimensionless distance parameter y^+ with ratio of wall viscosity to laminar sublayer viscosity and Reynolds' number is obtained. A comparison between the equation developed in the paper and the Nusselt empirical equation, including the constants evaluated by Dittus and Boelter, is made.

NOMENCLATURE

The following nomenclature is used in the paper:

- a = constant in empirical equation for viscosity
- A = area perpendicular to heat flow, ft²
- A_0 = inside surface area of pipe, ft²
- b = exponent in empirical equation for viscosity
- c_p = unit heat capacity of fluid, Btu/lb deg F
- C = constant
- D = diameter of pipe, ft
- f = friction factor for flow in pipe, dimensionless
- f_c = unit thermal conductance between pipe and fluid, Btu per (hr) (sq ft) (deg F)
- g = gravitational constant, ft/hr sec
- k = thermal conductivity of fluid, Btu per (hr) (ft²) (deg F/ft)
- l = Prandtl mixing length, ft
- \ln = natural logarithm
- m = exponent of Re in empirical heat-transfer Equation [2]; also subscript indicating mean temperature
- n = exponent of Pr in Equation [2]; also subscript denoting location of thermal resistance
- q = radial rate of heat transfer, Btu per hr
- q_0 = radial rate of heat transfer through pipe wall, Btu/hr
- r = distance from center of pipe to any point, ft
- r_0 = radius of pipe, ft
- t = temperature at any point y , deg F
- t' = fluctuating component of temperature, deg F
- t_1 = temperature at outer edge of laminar sublayer, deg F
- t_w = temperature of pipe wall, deg F
- u = average axial velocity of flow at any point y , fps

- u_1 = velocity at edge of laminar sublayer, fps
- u' = fluctuating component of axial velocity, fps
- u_{\max} = average axial velocity at center of pipe, fps
- u_{mean} = mean velocity of flow based on rate of discharge and pipe area, fps

$$u^+ = \text{dimensionless "velocity" parameter} = \frac{u}{\sqrt{\frac{\tau_0}{\rho}}}$$

- v' = fluctuating component of velocity perpendicular to axis of pipe, fps

- y = distance from wall, ft

- y_1 = distance from wall to outer edge of laminar sublayer, ft

- y_2 = distance from wall to outer edge of buffer layer, ft

$$y^+ = \text{dimensionless "distance" parameter} = \sqrt{\frac{\tau_0}{\rho}} y / \nu$$

- y_1^+ = dimensionless parameter fixing dimensions of laminar

$$\text{sublayer} = \sqrt{\frac{\tau_0}{\rho}} y_1 / \nu$$

- y_2^+ = dimensionless parameter fixing dimensions of buffer

$$\text{layer} = \sqrt{\frac{\tau_0}{\rho}} y_2 / \nu$$

- Δt = difference between temperature of pipe wall and any point y , deg F

- Δt_{\max} = difference between temperature of pipe wall and center of pipe, deg F

- Δt_{mean} = difference between temperature of pipe wall and average (mixed) temperature of fluid, deg F

- ϵ = eddy diffusivity, ft²/sec

- γ = weight density of fluid, lb/cu ft

- κ = Kármán constant = 0.4

- μ = viscosity of fluid, lb-sec/ft²

- μ_1 = viscosity of fluid at temperature t_1 , lb-sec/ft²

- μ_f = mean fluid viscosity in laminar sublayer, lb-sec/ft²

- μ_w = viscosity of the fluid at the temperature t_w , lb-sec/ft²

- ν = kinematic viscosity of fluid, ft²/sec

- ν_1 = kinematic viscosity of fluid at temperature t_1 , ft²/sec

- ξ = function which fixes velocity at edge of laminar sublayer

- ρ = density of fluid, (lb sec²)/ft⁴

- τ = unit shear at any point y , lb/ft²

- τ_0 = unit shear at wall, lb/ft²

- ϕ = function

$$Nu = \text{Nusselt's modulus} = \frac{f_c D}{k}$$

- Nu_1 = Nusselt's modulus of laminar sublayer

- Nu_2 = Nusselt's modulus of buffer layer

- Nu_3 = Nusselt's modulus of turbulent core

- Nu_n = Nusselt's modulus of any of three fluid layers

- Pr = Prandtl's modulus = $\mu c_p g / k$

- Pr_1 = Prandtl's modulus at temperature t_1

- Pr_m = Prandtl's modulus at mean (mixed) fluid temperature

$$Re = \text{Reynolds' modulus} = \frac{u_{\text{mean}} D \rho}{\mu}$$

- Re_m = Reynolds' modulus at mean (mixed) fluid temperature

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

INTRODUCTION

Application of the theory of similarity to the differential equations of heat transfer and fluid flow yields the result that for heat transfer from a fluid to a solid boundary in similar systems a relation

$$Nu = \phi(Re, Pr) \dots \dots \dots [1]$$

will exist. The function ϕ may be formulated from experimental data and, for purposes of design, an empirical expression of the form

$$Nu = CRe^m Pr^n \dots \dots \dots [2]$$

is normally employed.

Reynolds (1),³ and later Prandtl (2), G. I. Taylor (3), and von Kármán (4) have attempted to deduce the form of the function ϕ analytically for turbulent flow by comparing the exchange of momentum and of heat. Reynolds dealt with a purely turbulent system in which exact similarity between temperature and velocity fields was postulated (i.e., $Pr = 1$). Prandtl and G. I. Taylor extended the analogy to other values of the Prandtl modulus Pr by introducing a concept which included a laminar sublayer and a turbulent core. Von Kármán defined a buffer layer (of particular characteristics) which was located between the laminar layer and the turbulent core.

An attempt is made by the authors of this paper to provide an extension of the ideal system of von Kármán. In this ideal system, the resistances to heat transfer from a solid boundary to a fluid consist of the following:

- 1 A laminar sublayer in which viscous forces predominate. The heat is transferred through this sublayer by conduction only.
- 2 A "buffer" layer in which both viscous and eddy forces are important and through which heat is transferred by both thermal conduction and eddy diffusion.
- 3 A core of fluid in which eddy forces predominate. The heat is transferred in the core by eddy motion only.

The rate of heat transfer per unit area and the unit shear at any point in the system may be expressed as follows:

	Unit shear ⁴	Rate of heat transfer per unit area
Laminar sublayer	$\frac{\tau}{\rho} = \nu \frac{du}{dy}$	$\frac{q}{A} = -k \frac{dt}{dy}$
Buffer layer	$\frac{\tau}{\rho} = (\nu + \epsilon) \frac{du}{dy}$	$\frac{q}{Ac_p \gamma} = -\left(\frac{\nu}{Pr} + \epsilon\right) \frac{dt}{dy}$
Turbulent core	$\frac{\tau}{\rho} = \epsilon \frac{du}{dy}$	$\frac{q}{Ac_p \gamma} = -\epsilon \frac{dt}{dy}$

³ Numbers in parentheses refer to the Bibliography at end of paper.

⁴ See Note 1 of Appendix.

TABLE 1 DEFINITION OF IDEAL SYSTEM—ISOTHERMAL HEAT TRANSFER IN A CIRCULAR PIPE

	EXPERIMENTAL VELOCITY DISTRIBUTION	SHEAR	SHEAR EQUATION	EDDY DIFFUSIVITY	RATE OF HEAT FLOW	$\frac{q}{A}$	TEMPERATURE DROP	INTEGRATED FORM OF Δt	THERMAL RESISTANCES IN DIMENSIONLESS FORM
LAMINAR SUBLAYER	$u^+ = y^+$ $0 \leq y^+ \leq 5$	$\tau = \tau_0$	$\frac{\tau}{\rho} = \nu \frac{du}{dy}$	$\epsilon = 0$	$\frac{q}{A} = -k \frac{dt}{dy}$	$\frac{q}{A}$	$\Delta t = \frac{q}{A} \int_0^{y^+} \frac{dy}{\nu}$	$\Delta t = \frac{q}{A} \frac{y^+}{\nu}$	$\frac{1}{Nu} = \frac{y^+}{Re} \frac{1}{f}$
BUFFER LAYER	$u^+ = -3.05 + 5.00 \ln y^+$ $5 \leq y^+ \leq 30$	$\tau = \tau_0$	$\frac{\tau}{\rho} = (\nu + \epsilon) \frac{du}{dy}$	$\epsilon = \left(\frac{\tau}{\rho}\right)^2 \left(\frac{\nu}{Pr} + \epsilon\right)$	$\frac{q}{Ac_p \gamma} = -\left(\frac{\nu}{Pr} + \epsilon\right) \frac{dt}{dy}$	$\frac{q}{A}$	$\Delta t = \frac{q}{Ac_p \gamma} \int_0^{y^+} \left(\frac{\nu}{Pr} + \epsilon\right) dy$	$\Delta t = \frac{q}{Ac_p \gamma} \frac{1}{f} \ln \left(1 + \frac{Pr}{f} \left(\frac{y^+}{\nu} - 1\right)\right)$	$\frac{1}{Nu} = \frac{1}{Re} \frac{1}{f} \ln \left(1 + \frac{Pr}{f} \left(\frac{y^+}{\nu} - 1\right)\right)$
TURBULENT CORE	$u^+ = 5.5 + 2.5 \ln y^+$ $y^+ \geq 30$	$\tau = \tau_0$	$\frac{\tau}{\rho} = \epsilon \frac{du}{dy}$	$\epsilon = \frac{1}{4} \left(\frac{\tau}{\rho}\right)^2 \left(\frac{\nu}{Pr} + \epsilon\right)$	$\frac{q}{Ac_p \gamma} = -\epsilon \frac{dt}{dy}$	$\frac{q}{A} \left(1 - \frac{1}{Re}\right)$	$\Delta t = \frac{q}{Ac_p \gamma} \int_0^{y^+} \epsilon dy$	$\Delta t = \frac{q}{Ac_p \gamma} \frac{1}{f} \ln \frac{Re}{f}$	$\frac{1}{Nu} = \frac{1}{Re} \frac{1}{f} \ln \frac{Re}{f}$

A = AREA PERPENDICULAR TO HEAT FLOW AT ANY y
 f = FRICTION FACTOR
 k = THERMAL CONDUCTIVITY OF THE FLUID
 l = PRANDTL MIXING LENGTH
 q = RATE OF HEAT TRANSFER (RADIAL) AT ANY y
 r_0 = RADIUS OF PIPE
 t = TEMPERATURE OF FLUID AT ANY POINT y
 u = VELOCITY AT ANY POINT y
 y = DISTANCE FROM THE WALL
 y_1 = DISTANCE FROM WALL TO EDGE OF LAMINAR SUBLAYER
 y_2 = DISTANCE FROM WALL TO EDGE OF BUFFER LAYER
 γ = UNIT HEAT CAPACITY OF FLUID
 ϵ = RATE OF HEAT TRANSFER (RADIAL) AT $y = 0$
 $A_s = 2\pi r_0 l$ = SURFACE AREA OF PIPE
 l = LENGTH OF PIPE

ϵ = EDDY DIFFUSIVITY
 $\epsilon = K \text{ KÁRMÁN CONSTANT} = 0.40$
 ν = KINEMATIC VISCOSITY OF FLUID
 ρ = DENSITY OF FLUID
 τ = SHEAR AT ANY POINT y
 τ_0 = SHEAR AT WALL
 γ = UNIT WEIGHT OF FLUID
 Pr = PRANDTL'S MODULUS
 $u^+ = \frac{u}{\sqrt{\tau_0/\rho}}$
 $y^+ = \frac{\sqrt{\tau_0/\rho}}{\nu} y$
 $y_1^+ = \frac{\sqrt{\tau_0/\rho}}{\nu} y_1$
 $y_2^+ = \frac{\sqrt{\tau_0/\rho}}{\nu} y_2$
 $Re = \frac{u_m r_0}{\nu}$

For $Pr = 1$ the temperature and the velocity fields are similar.

The evaluation of the unit shear and the rate of heat transfer require a knowledge of the velocity gradient $\left(\frac{du}{dy}\right)$ at every point in the system.

"ISOTHERMAL" HEAT TRANSFER IN A CIRCULAR PIPE

For circular pipes, the experimentally determined velocity distribution may be correlated (7) by plotting the parameter

$$u^+ = \frac{u}{\sqrt{\tau_0/\rho}} \quad \text{against} \quad y^+ = \frac{\sqrt{\tau_0/\rho}}{\nu} y$$

One curve is obtained for all magnitudes of Reynolds' modulus, Re . This plot is shown in Fig. 1.

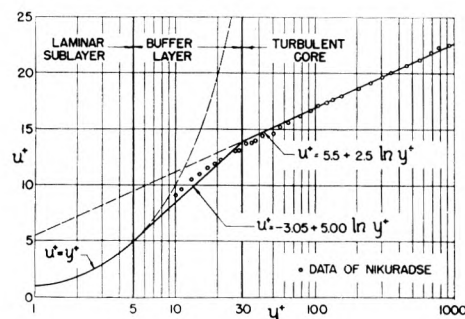


FIG. 1 VELOCITY DISTRIBUTION ACROSS A PIPE, REPRESENTED BY MEANS OF GENERALIZED COORDINATES

The following equations express the velocity distribution in the three fluid layers:

Laminar sublayer	$(0 \leq y^+ \leq 5)$	$u^+ = y^+$
Buffer layer	$(5 \leq y^+ \leq 30)$	$u^+ = -3.05 + 5.00 \ln y^+$
Turbulent core	$(y^+ \geq 30)$	$u^+ = 5.5 + 2.5 \ln y^+$

These equations allow the determination of $\left(\frac{du}{dy}\right)$ at any point in the flow system. The thermal resistance of each of the fluid layers may be calculated from the equations relating u^+ and y^+ .

The necessary operations are given in Table 1. The following specifications of the ideal system are to be added to those listed:

- 1 All fluid physical properties are independent of temperature; i.e., "isothermal" heat transfer is postulated.

2 The shear is constant in both the laminar sublayer and buffer layer, Fig. 2.

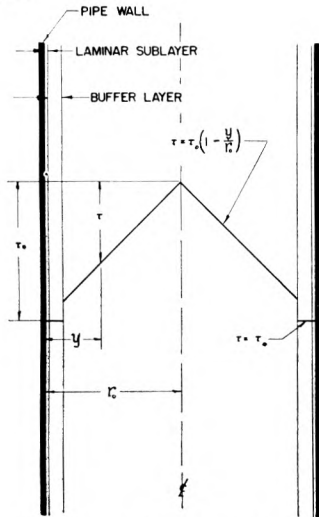


FIG. 2 UNIT-SHEAR DISTRIBUTION ACROSS PIPE
(As employed in ideal system presented in this paper.)

3 The rate of radial heat flow per unit area is constant except in the turbulent region where it varies linearly to zero at the center. The postulate may be shown to be very nearly the case by performing a heat balance on a differential annulus of fluid.

4 The mixing length l , in the turbulent region is defined (8) by the equation

$$l = \kappa y \sqrt{1 - y/r_0}$$

Experimental evidence reveals that this equation does not represent the facts. However, Prandtl (9), in effect,⁵ by making the same idealization, adequately represented the velocity distribution in a pipe.

5 The velocity gradient $\left(\frac{du}{dy}\right)$ is not zero at the center of the pipe. This is a well-known weakness of all present-day logarithmic expressions for velocity distribution.

From Table 1 the total resistance to heat transfer may be obtained

$$\frac{1}{Nu} = \sum_1^3 \frac{1}{Nu_n}$$

Using values of $y_1^+ = 5$ and $y_2^+ = 30$, which are reasonable from an inspection of the data in Fig. 1, the final result is

$$\frac{Nu}{Re Pr} = \frac{\sqrt{\frac{f}{8}}}{5 \left[Pr + \ln(1 + 5Pr) + 0.5 \ln \frac{Re}{60} \sqrt{\frac{f}{8}} \right]} \dots [3]$$

This expression for $\frac{Nu}{Re Pr}$ is based on the temperature difference between the pipe wall and the center of the pipe. Experimental results are always based on an average fluid temperature. Thus, to compare Equation [3] with experimental values, the right side should be divided by the ratio

$$\frac{\Delta t_{\text{mean}}}{\Delta t_{\text{max}}} \dots [4]$$

The temperature-difference ratio expressed in Equation [4] may be determined analytically from Equation [3].

The thermal resistance from the wall to any point y , located in the turbulent core, is given by

⁵ See Note 2 of Appendix.

$$\frac{5}{Re Pr} \sqrt{\frac{8}{f}} \left(Pr + \ln(1 + 5Pr) + 0.5 \ln \frac{Re}{60} \sqrt{\frac{f}{8}} \right) \dots [5]$$

The resistance to the center of the pipe is

$$\frac{5}{Re Pr} \sqrt{\frac{8}{f}} \left(Pr + \ln(1 + 5Pr) + 0.5 \ln \frac{Re}{60} \sqrt{\frac{f}{8}} \right) \dots [6]$$

Thus the temperature distribution in the turbulent core is given by

$$\frac{\Delta t}{\Delta t_{\text{max}}} = \left(\frac{Pr + \ln(1 + 5Pr) + 0.5 \ln \frac{Re}{60} \sqrt{\frac{f}{8}}}{Pr + \ln(1 + 5Pr) + 0.5 \ln \frac{Re}{60} \sqrt{\frac{f}{8}}} \right) \dots [7]$$

A plot of the temperature distribution for various magnitudes of the Prandtl modulus Pr at Reynolds' modulus Re equal to 10,000 is shown in Fig. 3.

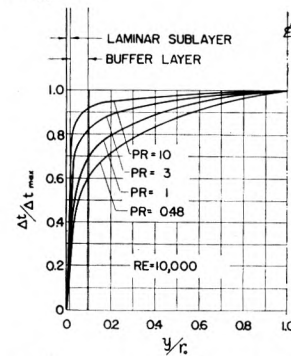


FIG. 3 PREDICTED TEMPERATURE DISTRIBUTION ACROSS PIPE SECTION FOR VARIOUS MAGNITUDES OF PRANDTL MODULUS AT REYNOLDS' NUMBER = 10,000
(Prediction is based upon Equation [7].)

The velocity distribution (10) is given by the equation

$$\frac{u}{u_{\text{max}}} = \left(\frac{5.5 + 2.5 \ln \frac{Re}{2} \sqrt{\frac{f}{8}}}{5.5 + 2.5 \ln \frac{Re}{2} \sqrt{\frac{f}{8}}} \right) \dots [8]$$

At a value of $Pr = 1$ the temperature distribution should be identical with the velocity distribution. Setting $Pr = 1$ in Equation [7]

$$\frac{\Delta t}{\Delta t_{\text{max}}} = \left(\frac{5.45 + 2.5 \ln \frac{Re}{2} \sqrt{\frac{f}{8}}}{5.45 + 2.5 \ln \frac{Re}{2} \sqrt{\frac{f}{8}}} \right) \dots [9]$$

which agrees favorably with Equation [8].

The mean temperature may be obtained graphically from the velocity and temperature distributions. Thus for an incompressible fluid

$$\frac{\Delta t_{\text{mean}}}{\Delta t_{\text{max}}} = \frac{\int_0^{r_0} \frac{u}{u_{\text{max}}} \times \frac{\Delta t}{\Delta t_{\text{max}}} r dr}{\int_0^{r_0} \frac{u}{u_{\text{max}}} \times r dr} \dots [10]$$

Curves of this expression for several magnitudes of Re are represented in Fig. 4. Inclusion of the effect of compressibility within the range of magnitudes employed by the experimenters quoted in this paper yielded results which did not differ appreciably from

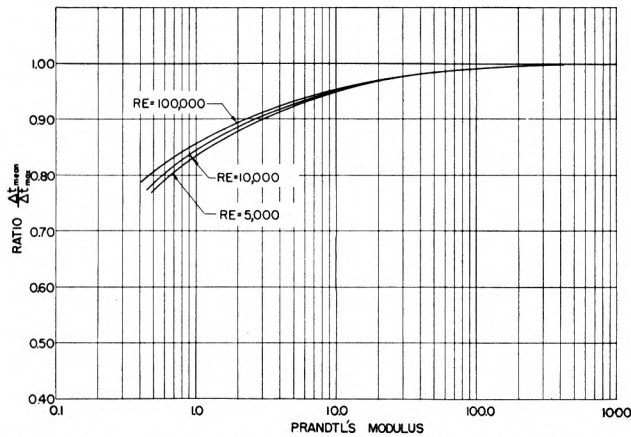


FIG. 4 RATIO OF MEAN TO MAXIMUM TEMPERATURE DIFFERENCES AS A FUNCTION OF PRANDTL MODULUS FOR REYNOLDS' NUMBERS = 5000, 10,000, AND 100,000 (Refer to Equation [10].)

those of Equation [10]. Thus, for comparison with experimental data, Equation [3] becomes

$$\frac{Nu}{Re Pr} = \frac{\sqrt{\frac{f}{8}} \frac{\Delta t_{max}}{\Delta t_{mean}}}{5 \left[Pr + \ln(1 + 5 Pr) + 0.5 \ln \frac{Re}{60} \sqrt{\frac{f}{8}} \right]} \quad [11]$$

Plots of this expression for various magnitudes of Re are shown

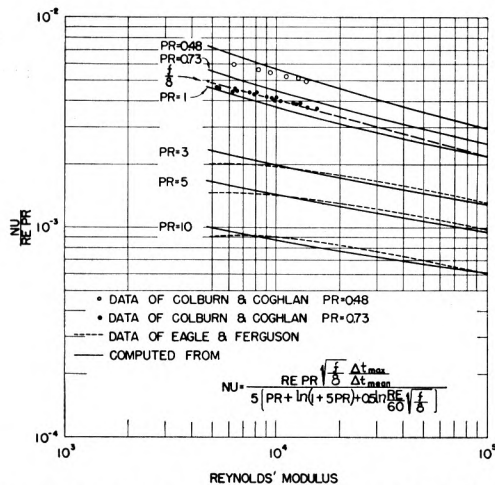


FIG. 5 EXPERIMENTAL AND PREDICTED MAGNITUDES OF $Nu/(Re Pr)$ AS FUNCTION OF REYNOLDS' MODULUS FOR DIFFERENT MAGNITUDES OF PRANDTL MODULUS

(Predicted magnitudes apply to ideal isothermal heat-transfer system.)

in Fig. 5, as well as the data of Eagle and Ferguson (11) and Colburn and Coghlan (12). The correlation is seen to be good except at low values of Re . This discrepancy may be due in part to the fact that, since thermal calming sections were not used, the temperature distribution considered in this analysis was not attained until the fluid traversed a considerable distance in the test section. This distance would be shorter the higher the magnitude of the unit thermal conductance. Thus better correlation at high Reynolds' numbers may be expected.

The lower limit of $\frac{Nu}{Re Pr}$ is that given by Equation [3], in which case the temperature is, in effect, considered constant across the pipe section. Calculation reveals that the experimental points

of Colburn and Coghlan fall between this lower limit and that given by Equation [11].

Equation [11] indicates that $\frac{Nu}{Re Pr}$ is directly proportional to $\sqrt{\frac{f}{8}}$ with another term involving the friction factor in the denominator.

For $Pr = 1$, Equation [11] becomes

$$\frac{Nu}{Re Pr} = \frac{\sqrt{\frac{f}{8}} \frac{\Delta t_{max}}{\Delta t_{mean}}}{5 \left[1 + \ln 6 + 0.5 \ln \frac{Re}{60} \sqrt{\frac{f}{8}} \right]} \quad [12]$$

As shown in Table 2, the coefficient of $\sqrt{\frac{f}{8}}$ is almost numerically equal to $\sqrt{\frac{f}{8}}$ so that the following relation results

$$\frac{Nu}{Re Pr} \cong \frac{f}{8} \quad [13]$$

This result was obtained by Reynolds (1) for the ideal system in which the heat transfer was accomplished by eddy diffusion only.

TABLE 2 COMPARISON OF MAGNITUDES INVOLVED IN EQUATION [12]

Re	$\sqrt{\frac{f}{8}}$	$5 \left(1 + \ln 6 + 0.5 \ln \frac{Re}{60} \sqrt{\frac{f}{8}} \right)$
5000	14.3	15.0
10000	15.8	16.6
100000	21.4	22.0

NONISOTHERMAL HEAT TRANSFER IN A CIRCULAR PIPE

The previous analysis is applicable only to "isothermal" heat transfer, since variations of viscosity and other fluid properties with temperature were not considered. The data of Colburn and Coghlan and Eagle and Ferguson are comparable to these conditions for, in the former, variations of viscosity were small and, in the latter, all results were extrapolated to conditions of zero heat transfer. On the other hand, the analysis of heat transfer in fluids which possess a highly variable viscosity requires the definition of another ideal system. In particular, the boundary of the laminar sublayer for nonisothermal flow requires investigation. The dimensionless variable, y_1^+ , which fixes the outer boundary of the laminar sublayer will be redefined as a point function (exclusive of the unit-shear term which is defined as constant). Thus

$$y_1^+ = \frac{\sqrt{\frac{\tau_0}{\rho}} y_1}{\nu_1} \quad [14]$$

where ν_1 is the kinematic viscosity of the fluid at the edge of the laminar sublayer and τ_0 is the shear at the wall under nonisothermal conditions. Evaluation of the thermal resistance of the laminar sublayer yields the expression

$$\frac{1}{Nu_1} = \frac{y_1^+ \sqrt{\frac{8}{f}} Pr_1}{Re_m Pr_m} \quad [15]$$

Without introducing too great an error, the remainder of the fluid system may be defined as possessing a constant viscosity which is fixed by the mean fluid temperature. The expression for $\frac{Nu}{Re Pr}$ then becomes

$$\frac{Nu}{Re_m Pr_m} = \sqrt[3]{\frac{f}{8} \frac{\Delta t_{\max}}{\Delta t_{\text{mean}}}} \quad [16]$$

$$y_1^+ \left[Pr_1 + \ln \left(1 + Pr_m \left\{ \frac{30}{y_1^+} - 1 \right\} \right) \right] + \frac{2.5}{y_1^+} \ln \frac{Re_m}{60} \sqrt[3]{\frac{f}{8}} \quad [16]$$

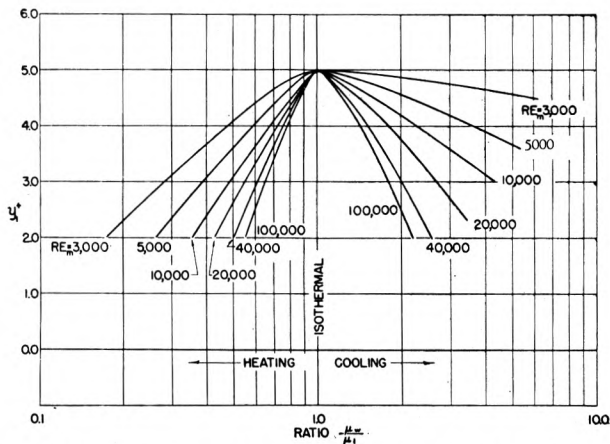
Examination reveals that Equation [11] is a special case of Equation [16]. A large variation of viscosity across the laminar sublayer should cause variations in y_1^+ , which fixes its boundary. To determine this variation, the data of Morris and Whitman (13) were substituted into Equation [16]. Excellent correlation resulted, y_1^+ being a function of Reynolds' modulus, and the ratio

$\frac{\mu_w}{\mu_1}$ where μ_w = the viscosity of the fluid at the wall temperature and μ_1 = the viscosity of the fluid at the temperature existing at the edge of the laminar sublayer.⁶

The results of the computations are shown in Fig. 6, and are tabulated in Table 3. Using the curves of Fig. 6 and the experi-

TABLE 3 MAGNITUDES OF y_1^+

$\frac{\mu_w}{\mu_1}$	Re_m 3000	Re_m 5000	Re_m 10000	Re_m 20000	Re_m 40000	Re_m 100000
0.10
0.20	2.30
0.30	3.12	2.34
0.40	3.70	3.07	2.40
0.50	4.10	3.60	3.09	2.60	2.00	..
0.60	4.43	4.03	3.62	3.34	2.90	2.50
0.70	4.68	4.40	4.10	3.90	3.62	3.40
0.80	4.82	4.70	4.50	4.40	4.25	4.10
0.90	4.95	4.90	4.80	4.75	4.70	4.65
1.00	5.00	5.00	5.00	5.00	5.00	5.00
1.50	4.95	4.80	4.60	4.35	4.00	3.75
2.00	4.89	4.52	4.20	3.75	3.00	2.50
3.00	4.75	4.18	3.60	2.70
4.00	4.65	3.90	3.15
5.00	4.57	3.70
6.00	4.50
7.00

FIG. 6 MAGNITUDE OF y_1^+ AT INTERFACE BETWEEN LAMINAR SUB-LAYER AND BUFFER LAYER AS A FUNCTION OF RATIO OF WALL TO INTERFACE VISCOSITIES, REYNOLDS' NUMBER AS PARAMETER

mental conditions of Colburn and Coghlan, Eagle and Ferguson, Morris and Whitman, and J. F. D. Smith (14), magnitudes of Nu were computed. The magnitude of the predicted Nusselt modulus employing Equation [16] was plotted against the experimental magnitudes in Fig. 7.

The friction factors used in the foregoing correlation were obtained from the isothermal friction-factor curve⁷ at a Reynolds number computed using the mean laminar sublayer viscosity μ_f instead of the viscosity at the mean fluid temperature μ_m . This viscosity (17) may be found from the relation

⁶ See Note 3 of Appendix.

⁷ See Note 4 of Appendix.

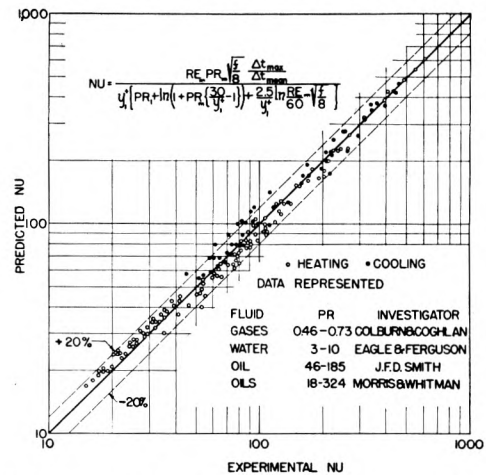


FIG. 7 PREDICTED MAGNITUDE OF NUSSELT MODULUS PLOTTED AS FUNCTION OF EXPERIMENTAL MAGNITUDES OF NUSSELT MODULUS, FOR HEATING AND COOLING, AND FOR RANGE OF PRANDTL MODULUS FROM 0.46 TO 324

(Refer to Equation [16].)

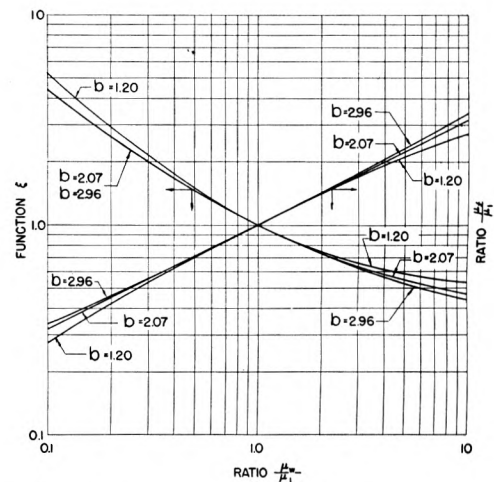
$$\mu_f = \frac{1}{y_1} \int_0^{y_1} \mu dy \dots \dots \dots [17]$$

If $\mu = \frac{a}{t^b}$ (an empirical expression of wide application)—then

$$\mu_f = \mu_1 \left\{ \frac{\left(\frac{\mu_1}{\mu_w} \right)^{\frac{1-b}{b}} - 1}{(1-b) \left[\left(\frac{\mu_1}{\mu_w} \right)^{\frac{1}{b}} - 1 \right]} \right\} \dots \dots \dots [18]$$

A plot of this expression is shown in Fig. 8.

A comparison of the predicted value of $\sqrt[3]{\frac{f}{8}}$ as compared with the measured values of J. F. D. Smith is shown in Fig. 9. The check is usually within 6 per cent which is sufficiently accurate for

FIG. 8 FUNCTION ξ PLOTTED IN TERMS OF RATIO OF WALL TO INTERFACE VISCOSITIES FOR VARIOUS MAGNITUDES OF VISCOSITY EXPONENT b ; RATIO OF MEAN LAMINAR SUB-LAYER VISCOSITY TO INTERFACE VISCOSITY AS A FUNCTION OF RATIO OF WALL TO INTERFACE VISCOSITIES

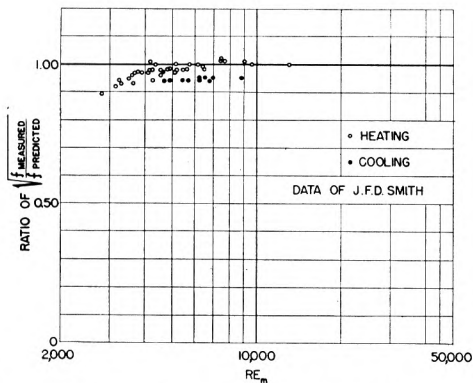


FIG. 9 RATIO OF THE SQUARE ROOT OF THE MEASURED TO THE PREDICTED FRICTION FACTOR FOR NONISOTHERMAL FLOW AS A FUNCTION OF REYNOLDS' MODULUS

the correlation shown in Fig. 7. Rohonczi (16) suggests a method of presenting the nonisothermal friction factor in terms of Reynolds' modulus evaluated at the wall temperature. This method results in over-correcting the friction factor.

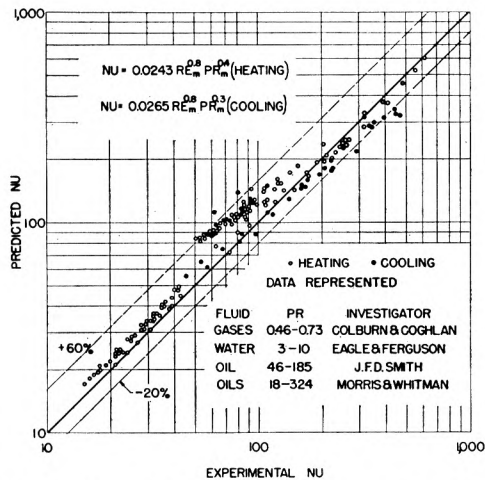


FIG. 10 PREDICTED VERSUS EXPERIMENTAL MAGNITUDES OF NUSSELT MODULUS
(Predictions in terms of empirical Equations [19] and [20].)

A practical limit to the correlation with the available experimental data exists. This limitation is illustrated as follows:

Investigators	Run	Re_m	Pr_m	$\frac{\mu_w}{\mu_1}$	Nu
J. F. D. Smith	13 (cooling)	5410	103	4.0	71.1
Morris and Whitman	F 16	5460	99	3.4	98.0

Thus for practically identical experimental conditions the two investigators reported a difference in Nusselt's modulus of 40 per cent. Some of this difference is explained by the fact that Morris and Whitman used commercial pipe, while J. F. D. Smith used smooth pipe. Even after making allowance for this fact the values of Nu differ by about 30 per cent.

In order to compare the results from Equation [16] with the empirical equations (15).

$$Nu = 0.0243 Re_m^{0.8} Pr_m^{0.4} \text{ (heating)} \dots \dots \dots [19]$$

$$Nu = 0.0265 Re_m^{0.8} Pr_m^{0.3} \text{ (cooling)} \dots \dots \dots [20]$$

the same experimental values used in Fig. 7 are plotted against the values computed by Equations [19] and [20], and are shown

in Fig. 10. In Equations [19] and [20] the fluid properties are evaluated at the arithmetic mean fluid temperature.

Inspection of Equation [16] indicates that Nu should be practically independent of Pr for large magnitudes of the Prandtl modulus. However, variations of y_1^+ accompany the variations in viscosity which always occur in the experimental determination of Nu . These variations in y_1^+ give the apparent influence of Pr expressed by Equations [19] and [20].

The velocity existing at the edge of the laminar sublayer for nonisothermal conditions may be readily estimated. Since

$$\frac{\tau_0}{\rho} = \nu \frac{du}{dy}$$

and

$$u_1 = \int_0^{y_1} \frac{\tau_0 dy}{\mu}$$

which upon integration with

$$\mu = \frac{a}{b^y} \dots \dots \dots [21]$$

$$\text{becomes } \frac{u_1}{\sqrt{\frac{\tau_0}{\rho}}} = y_1^+ \left\{ \frac{1 - \left(\frac{\mu_1}{\mu_w} \right)^{\frac{b+1}{b}}}{(1+b) \left[1 - \left(\frac{\mu_1}{\mu_w} \right)^{\frac{1}{b}} \right]} \right\} \dots \dots \dots [22]$$

or

$$\frac{u_1}{\sqrt{\frac{\tau_0}{\rho}}} = y_1^+ \xi$$

A plot of the function ξ is given in Fig. 8.

CONCLUSIONS

1 The thermal resistances involved in heat transfer from a boundary to a turbulently moving fluid have been segregated. An expression which correlates existing heat-transfer data for flow through tubes is proposed for the system in which the fluid properties are not functions of temperature.

2 The introduction of the concept of a variable y_1^+ , the utilization of an average viscosity rather than the viscosity at an average temperature, and the estimation of a nonisothermal friction factor make possible the prediction of unit thermal conductances to within ± 20 per cent for Reynolds' numbers up to 100,000 and magnitudes of the Prandtl modulus between 0.5 and 325.

3 An ideal system may be defined which will more accurately predict $Nu/(Re Pr)$ but the algebraic evaluation will be tedious. Among other operations, the variation of Nu with length (due to the changes in properties and of temperature distribution and the variation of q/A with radius) should be considered. Additional accurate experimental heat-transfer data will aid the analyst.

4 Finally, a more precise formulation of the laws governing the velocity distribution (18) will allow the definition of a more satisfactory ideal system for heat transfer.

Appendix

NOTE 1

Reynolds (5) conceived the flow at a point in a two-dimensional system to be composed of an average velocity u upon which are superimposed fluctuating components u' , v' . The unit shear due to the momentum transferred by these fluctuating components becomes

$$\tau = -\rho \overline{u'v'}$$

where the expression $\overline{u'v'}$ represents the time average of the instantaneous fluctuations u' and v' . Prandtl (6) assumed that the fluctuating velocities would be equal to the product of a "mixing" length l and the velocity gradient at the point. Thus the magnitude of the shear becomes

$$|\tau| = \rho l^2 \left(\frac{du}{dy} \right)^2$$

By analogy to the expression for shear in viscous flow

$$\tau = \rho \nu \frac{du}{dy}$$

an eddy "kinematic viscosity" ϵ may be defined as

$$\epsilon = l^2 \frac{du}{dy}$$

and

$$\tau = \rho \epsilon \frac{du}{dy}$$

Similar reasoning in the case of the transfer of heat yields the rate of heat transfer per unit area in turbulent flow as

$$\frac{q}{A} = c \gamma \rho \overline{v't'}$$

where v' and t' are the fluctuating components of velocity and temperature, respectively. Then if

$$t' = l \frac{dt}{dy}$$

$$v' = l \frac{du}{dy}$$

the rate of heat transfer per unit area in turbulent flow becomes

$$q/A = c_p \gamma \epsilon \frac{dt}{dy}$$

The identity of the eddy diffusivity ϵ for heat transfer and momentum transfer is known as Reynolds' analogy.

NOTE 2

Prandtl actually postulated a constant shear across the pipe and a mixing length $= \kappa y$. Thus

$$\frac{\tau}{\rho} = \frac{\tau_0}{\rho} = \epsilon \frac{du}{dy}$$

and

$$\epsilon = l^2 \frac{du}{dy}$$

$$\frac{\tau_0}{\rho} = \kappa^2 y^2 \left(\frac{du}{dy} \right)^2$$

from which

$$u = \frac{\sqrt{\tau_0/\rho}}{\kappa} \ln y + C$$

Exactly the same result is obtained if

$$\tau = \tau_0 \left(1 - \frac{y}{r_0} \right)$$

$$l = \kappa y \sqrt{1 - y/r_0}$$

for then

$$\frac{\tau}{\rho} = \frac{\tau_0}{\rho} \left(1 - \frac{y}{r_0} \right) = \kappa^2 y^2 \left(1 - \frac{y}{r_0} \right) \left(\frac{du}{dy} \right)^2$$

whence

$$\frac{\tau_0}{\rho} = \kappa^2 y^2 \left(\frac{du}{dy} \right)^2$$

NOTE 3

The temperature at the edge of the laminar sublayer was estimated by computing the thermal resistance of the laminar sublayer, buffer layer, and core. Then as a first approximation

$$\frac{t_w - t_1}{\Delta t_{\max}} = \frac{Pr_m}{\left[Pr_m + \ln(1 + 5Pr_m) + 0.5 \ln \frac{Re_m}{60} \sqrt{\frac{f}{8}} \right]}$$

Knowing t_1 an estimate of y_1^+ can be made.

Then more precisely

$$\frac{t_w - t_1}{\Delta t_{\max}} = \frac{Pr_1}{\left[Pr_1 + \ln \left(1 + Pr_m \left\{ \frac{30}{y_1^+} - 1 \right\} \right) + \frac{2.5}{y_1^+} \ln \frac{Re_m}{60} \sqrt{\frac{f}{8}} \right]}$$

NOTE 4

For predictions corresponding to the experimental data of Colburn and Coghlan, and Eagle and Ferguson, the friction data of Stanton and Pannell for smooth pipe were used.

For the J. F. D. Smith comparison, his own experimental magnitudes of f were used and for the Morris and Whitman comparison the following isothermal friction-factor curve (rough pipe) was utilized:

Re_m	$\sqrt{\frac{8}{f}}$
2000	12.0
5000	13.7
10000	14.9
30000	16.6
100000	18.3

ACKNOWLEDGMENT

The authors wish to express their thanks to Mr. John Longwell who performed many of the computations in this paper, and to the National Youth Administration for assistance given.

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Discussion

R. H. NORRIS.⁸ The results achieved in this paper would be of greater practical value, if the authors could choose one or more relatively simple equations as practical approximations to their complicated correlation. Discrepancies up to 60 per cent are shown to be produced by the simple Equations [19] and [20], heretofore in common use.

This paper is of course concerned with the relation between heat transfer and fluid friction, in terms of the parameter cu/k , only for conditions of turbulent flow. For viscous flow, however, there is also a relation between heat transfer and fluid friction, the existence of which may, heretofore, have been obscured by the derivation of the theoretical results for heat transfer independently from those for friction, and the arbitrary choice of the arithmetic mean for the temperature difference.

In the region of viscous flow, when the logarithmic mean is used for the temperature difference, the variable denoted as $(Nu)/(Re)(Pr)$ can be plotted versus Re , as shown in Fig. 3, of a recently published paper.⁹ When the ratio of pipe length L to the diameter D becomes sufficiently great, the value of $(Nu)/(Re)(Pr)$ becomes asymptotic to a straight line of slope -1 , the same slope that the curve of friction factor has in the viscous region;

thus
$$\frac{(Nu)}{(Re)(Pr)} = \frac{3.66}{(Re)(Pr)} \quad \text{for } (cpu_{\text{mean}}D^2)/(kL) < 4,$$
 and $f = 64 (Re)^{-1}$ so that
$$\frac{(Nu)}{(Re)(Pr)} = \frac{0.458(f/8)}{(Pr)} \quad \text{for } (cpu_{\text{mean}}D^2)/(kL) < 4.$$

For larger values of $(cpu_{\text{mean}}D^2)/(kL)$, with laminar flow, the slope becomes flatter, but this can be considered as a thermal "entrance effect," resulting from the sudden change in surface temperature at the start of the heated section of the duct. The viscous-flow-friction-factor curve would likewise flatten if the pipe length were sufficiently decreased and the calming section omitted.

⁸ General Engineering Laboratory, General Electric Company, Schenectady, N. Y. Jun. A.S.M.E.

⁹ "Laminar-Flow Heat-Transfer Coefficients for Ducts," by R. H. Norris and D. D. Streid, *Trans. A.S.M.E.*, vol. 62, August, 1940, pp. 525-533.

It should be noted that f , as defined by the authors, is 4 times the value sometimes given in the definitions of f by other authors.

E. S. SMITH.¹⁰ The writer wishes to ask the authors just what they would expect to see happen in the throat of a venturi tube in which the flow is at a sufficiently high Reynolds number that turbulence would be high in a long straight pipe of the same diameter and with the same fluid moving at the same velocity? The eddies should be powerfully suppressed in the venturi throat and the shear would be high near the wall of the throat since the velocity distribution is so nearly uniform. This question is raised in a paper by W. S. Pardoe,¹¹ in which his results can be explained by having the temperature of the water next to the wall under such conditions substantially equal to that of the ambient air. Now this does not seem to the writer to be reasonable, although he does not profess to be an expert on heat transfer. It would seem that there should be some gradient or drop of temperature from the air to the wall, assuming that the air temperature is higher.

TH. VON KÁRMÁN.¹² This paper is a valuable contribution to the solution of the problem of heat transfer. The authors give an excellent presentation of the similarity between heat and momentum transfer following the lines indicated in the writer's paper¹³ on the same subject. The new expression "buffer layer" for the transition layer between perfectly laminar and perfectly turbulent regions, introduced by the writer in order to explain the discrepancies between experiments and the theories of Prandtl and G. I. Taylor, is quite appropriate. The investigation is definitely carried further beyond the results given in the former paper¹³ by a more detailed analysis of the turbulent region and especially by consideration of the temperature changes in the cross section of the pipe. It is gratifying that, by a more exact analysis, the agreement between theory and experiments appears even closer than the writer has found.

AUTHORS' CLOSURE

For low magnitudes of the Prandtl modulus, the empirical expressions, Equations [19] and [20], or their equivalent, such as Colburn's j function (19),¹⁴ approximate the analytical relation with some accuracy. Fig. 11 of this closure illustrates the analytical equation plotted in terms of Colburn's j function and Fig. 12 reveals a comparison of Colburn's empirical equation, Reynolds', Prandtl's, and von Kármán's analogies, and Equation [11] of this paper. Below magnitudes of Prandtl's modulus equal to 10 the average slope of the analytical curve is approximately $-2/3$. However, at high magnitudes of Pr the slope of the analytical curve becomes -1 , which indicates that, for low rates of heat transfer (isothermal) and high magnitudes of Prandtl's modulus, Nusselt's modulus will be independent of Prandtl's modulus. Thus, the exponent of the Prandtl modulus in any empirical equations, such as Equation [19] or [20], should be a function of Pr and the rate of heat transfer. It appears to be fortuitous that a constant exponent yields a satisfactory correlation.

Several other modes of empirical correlation, using viscosities at temperatures other than the mixed mean temperature have

¹⁰ C. J. Tagliabue Mfg. Co., Brooklyn, N. Y. Mem. A.S.M.E.

¹¹ "Effect of Ambient Temperatures on the Coefficients of Venturi Meters," by W. S. Pardoe, *Trans. A.S.M.E.*, vol. 63, 1941, Figs. 5 and 10, p. 458.

¹² Director, Daniel Guggenheim Aeronautical Laboratory, California Institute of Technology, Pasadena, Calif. Mem. A.S.M.E.

¹³ "The Analogy Between Fluid Friction and Heat Transfer," by Th. von Kármán, *Trans. A.S.M.E.*, vol. 61, 1939, pp. 705-710.

¹⁴ Numbers (19) to (23) refer to Bibliography at the end of this closure; all other numbers in parentheses refer to the Bibliography at the end of the paper.

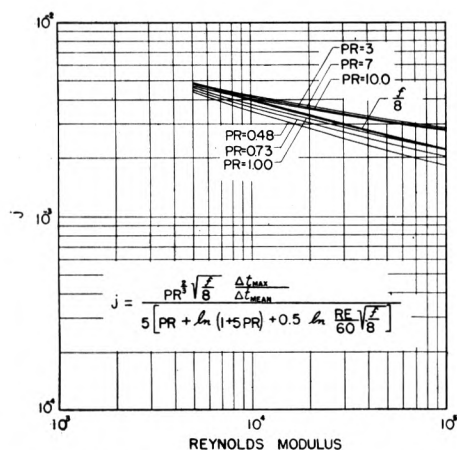


FIG. 11 PLOT OF COLBURN'S j FUNCTION $[Nu/(Re Pr)] Pr^{2/3}$ CALCULATED FROM EQUATION [11] FOR MAGNITUDES OF PRANDTL'S MODULUS BETWEEN 0.48 AND 10

(The friction factor over eight ($f/8$) is also presented as a function of Reynolds' modulus.)

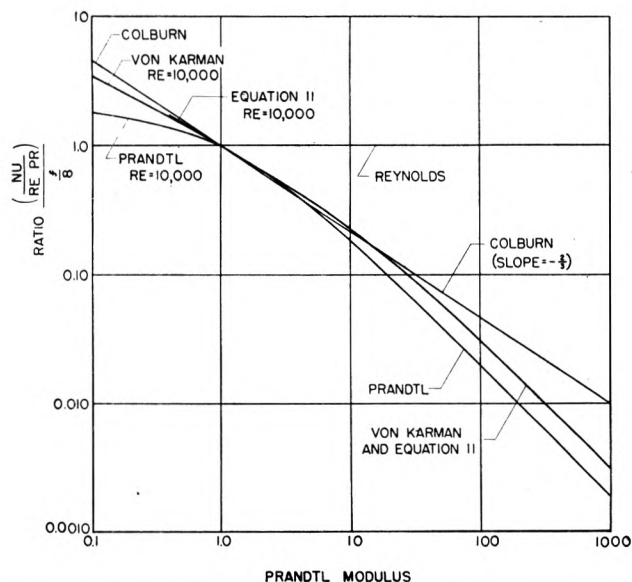


FIG. 12 COMPARISON OF REYNOLDS', PRANDTL'S, COLBURN'S, AND VON KÁRMÁN'S EQUATIONS FOR VARIOUS MAGNITUDES OF PRANDTL'S MODULUS FROM 0.1 TO 1000 FOR REYNOLDS' MODULUS = 10,000 (The magnitudes as computed from Equation [11] are also presented.)

been proposed (14, 17). Fair success in bringing experimental data together was attained by these methods so that Fig. 10 of the paper may leave the impression of overestimating the inaccuracy of empirical correlations. However, the same fundamental limitations apply to these later correlations as to those presented in the paper. Equations [19] and [20], or similar forms, are in current use in industrial design.

An important phenomenon which will cause scattering of data, particularly at low values of Prandtl's modulus, is the entrance effect mentioned in the body of the paper.

A uniform temperature distribution exists at the entrance to a heating section, which will change to a distribution such as is shown in Fig. 3 of the paper if the heating section is of sufficient length. If, however, the test section is short, it is possible that the equilibrium-temperature distribution will not be reached.

Application of Equation [11] to entrance sections will yield magnitudes of Nusselt's modulus which are too low (for instance, $\frac{\Delta T_{\text{mean}}}{\Delta T_{\text{max}}} = 1$ for uniform temperature distribution at entrance) because it applies only where the velocity and temperature distributions have been established and steady-state conditions obtain. Due to the transition conditions existing in the thermal quieting sections, the magnitudes of Nusselt's modulus have been found to exceed those which obtain after the temperature distribution has been established (20).

If the linear variation in unit shear in the buffer layer is utilized, rather than the constant wall-unit shear, the predicted magnitudes of $Nu/Re Pr$ for low magnitudes of Reynolds' and Prandtl's moduli fall below those shown in Fig. 5 of the paper, and approach closely the experimental points of Colburn and Coghlan (21).

Referring to E. S. Smith's comments, regarding nonisothermal flow in venturi meters, a difference in temperature between the ambient air and the fluid within the pipe will cause heat transfer to (or from) the fluid. Heat transfer will occur throughout the entrance length as well as in the venturi proper. This temperature difference will cause free convection in the entrance length, and will also change the viscosity of the fluid in the laminar sublayer in contrast to the magnitude corresponding to the mixed mean fluid temperature. These phenomena may influence the discharge coefficient of the venturi and account for the fact that the nonisothermal discharge-coefficient-Reynolds-number curve crosses the isothermal curve.

Dr. Theo. von Kármán's comments are indeed gratifying. Each correction as it is applied yields a prediction which more nearly approaches the best experimental results. The segregation of the resistances, due to each of the three fluid layers, has aided the authors in making further refinements to the isothermal-heat-transfer theory. The attendant success served as a stimulus toward the analysis presented (yet incomplete) for non-isothermal flow.

GENERAL COMMENTS

Four excellent articles (22, 23, 24, 25) on this subject have come recently to the attention of the authors. Mattioli (23) also utilizes y^+ as a point function. He considers the transfer of vorticity and momentum simultaneously.

The similarity of $Nu/Re Pr = f_c/Gc_p$, where G is the unit mass-flow rate, to the number of transfer units (N.T.U.) deserves attention in view of the move to assign a name to the former. In the tube-to-fluid exchanger, the N.T.U. = $(Nu/Re Pr)S/A$, where S is the tube area through which heat flows and A is the cross-sectional area of the pipe.

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- 24 "Über die Gesetzmäßigkeiten der Wärme- und der Stoffübertragung auf Grund der Strömungsvorgänge im Rohr," by E. Hofmann, *Forschung auf dem Gebiete des Ingenieurwesens*, Ausgabe A, Band 11, no. 4, July-Aug., 1940, pp. 159-169.
- 25 "Die Wärmeübertragung in turbulenten Reikungsschichten," by H. Reichardt, *Zeitschrift für Angewandte Mathematik und Mechanik*, Band 20, no. 6, December, 1940, pp. 297-328.

Effect of Ambient Temperatures on the Coefficients of Venturi Meters

By W. S. PARDOE,¹ PHILADELPHIA, PA.

PRIOR to 1930, the writer attempted to plot the coefficients of a large number of venturi tubes against Reynolds' number and found the results rather unsatisfactory. This would, of course, be easily explained on the basis of proportional roughness and, as there seemed to be no definition of this function, the results were considered most unsatisfactory.

During that period, on three occasions venturi tubes were sent to the University of Pennsylvania laboratory for calibration.

Each time the makers proceeded to construct a meter to fit the laboratory calibration. About 2 months later, when the combination of meter and venturi tube was tested in the laboratory with different temperature water, the coefficient was satisfactory on the flat part of the curve and at some point lower, about 3 fps throat velocity, but in between the coefficients varied as much as 0.5 per cent. This, of course, was all very confusing, as it was contradictory to any plot against Reynolds' number.

In 1931, the Simplex Valve and Meter Company presented the laboratory with an $8 \times 3\frac{3}{8}$ -in. bronze venturi meter. The results of tests on this meter for 46, 63, 69, and 74 F water are shown in Figs. 1 and 2. These are quite similar to other curves on venturi meters. It will be noticed that on the flat part of the curve the coefficient is 0.99 and at 3 fps throat velocity the coefficient is 0.969. Curves such as that for 46 F became known

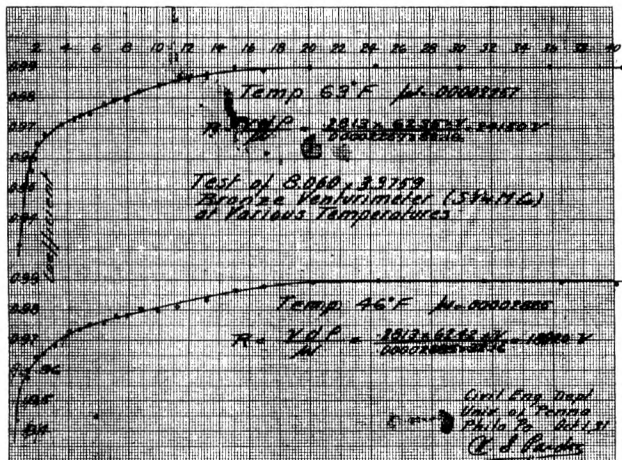


Fig. 1

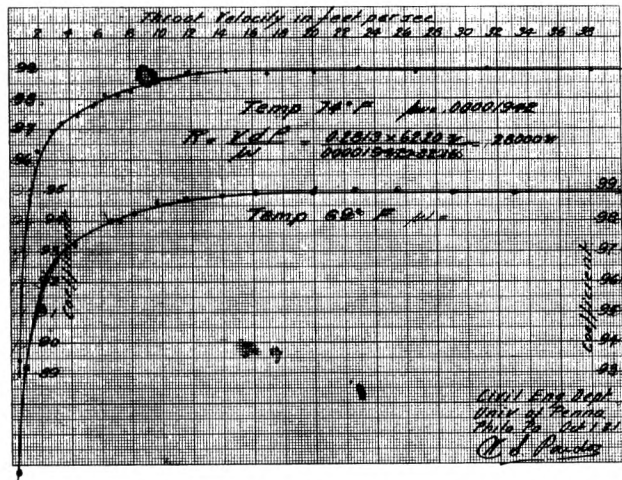


Fig. 2

FIGS. 1 AND 2 TEST OF 8.06-IN. BY 3.3759-IN. BRONZE VENTURI METER AT VARIOUS TEMPERATURES
(Courtesy of Simplex Valve and Meter Company.)

¹Professor, Department of Civil Engineering, University of Pennsylvania.

Contributed by Special Research Committee on Fluid Meters and presented at the Annual Meeting, New York, N. Y., December 2-6, 1940, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

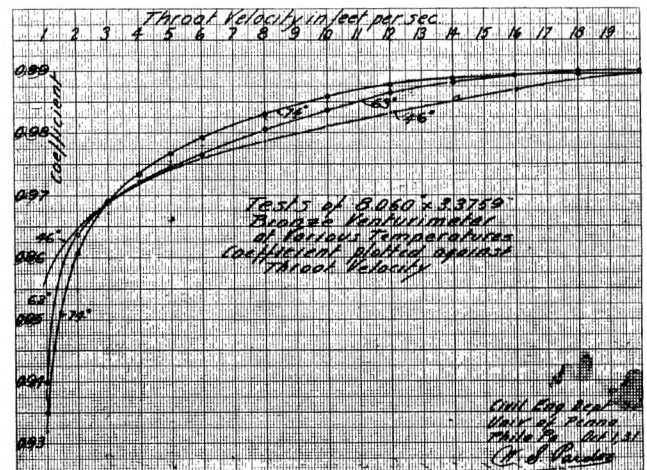


FIG. 3 TESTS OF VENTURI METER AT VARIOUS TEMPERATURES SHOWING COEFFICIENT PLOTTED AGAINST THROAT VELOCITY

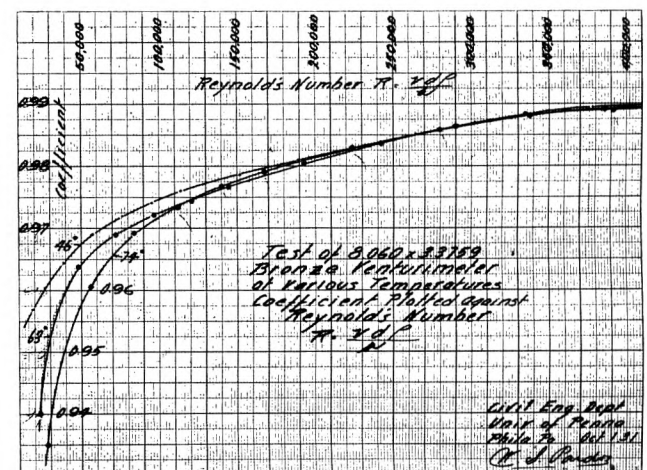


FIG. 4 TESTS OF VENTURI METER AT VARIOUS TEMPERATURES; COEFFICIENT PLOTTED AGAINST REYNOLDS' NUMBER

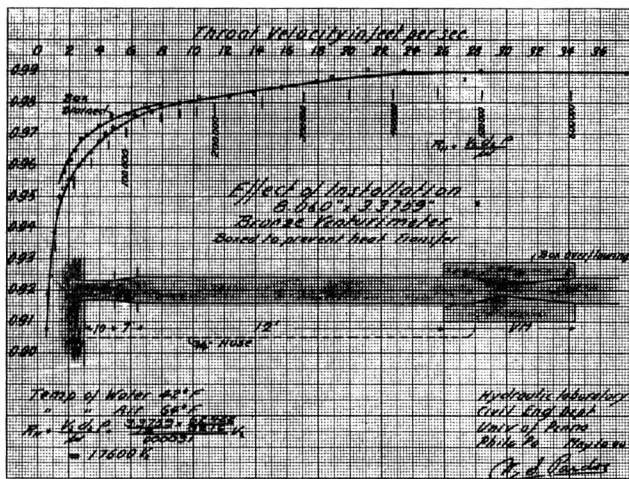


FIG. 5 EFFECT OF INSTALLATION OF VENTURI METER BOXED TO PREVENT HEAT TRANSFER; WATER TEMPERATURE 42 F

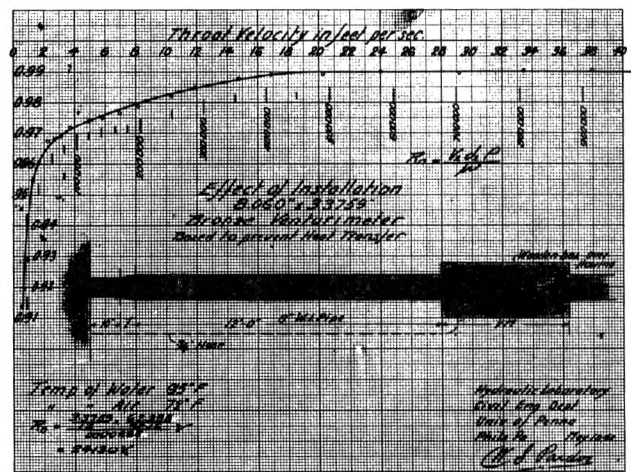


FIG. 8 EFFECT OF INSTALLATION OF VENTURI METER BOXED TO PREVENT HEAT TRANSFER; WATER TEMPERATURE 61.5 F

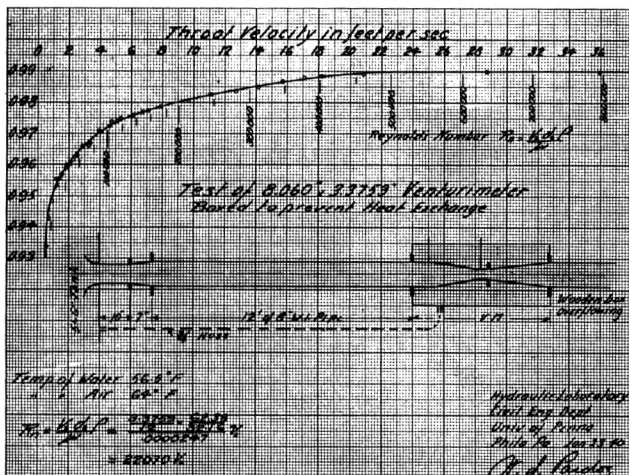


FIG. 6 TEST OF VENTURI METER, BOXED TO PREVENT HEAT TRANSFER; WATER TEMPERATURE 53 F

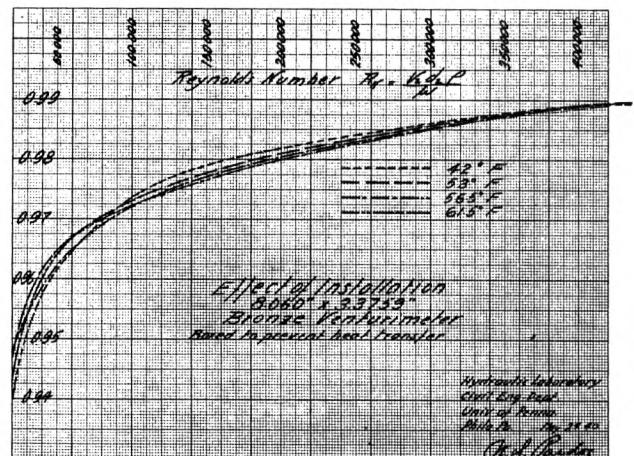


FIG. 9 TESTS OF FIGS. 5 TO 8, INCLUSIVE, PLOTTED AGAINST REYNOLDS' NUMBER

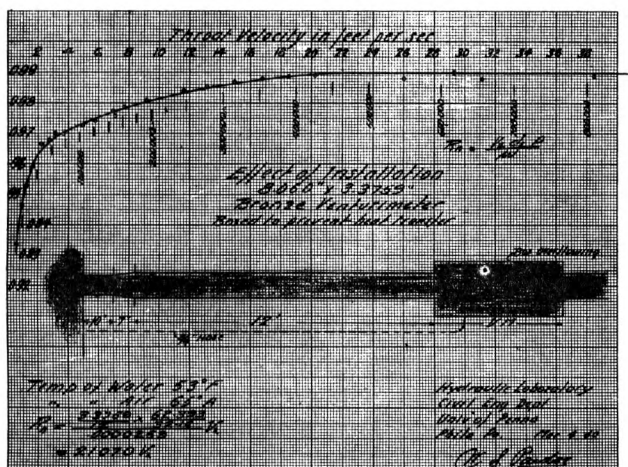


FIG. 7 EFFECT OF INSTALLATION OF VENTURI METER BOXED TO PREVENT HEAT TRANSFER; WATER TEMPERATURE 56.5 F

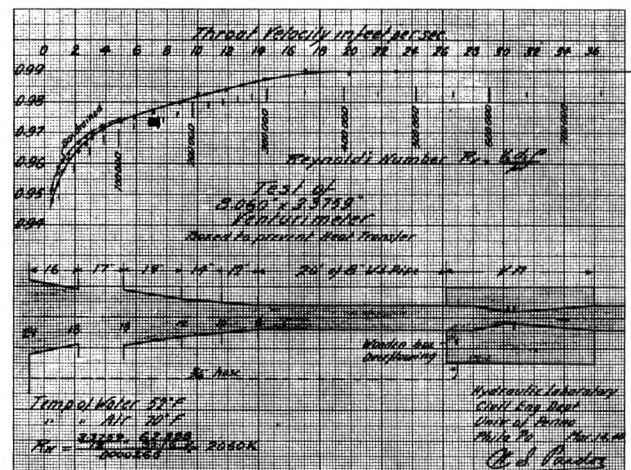


FIG. 10 TESTS MADE TO CHECK THE EFFECT OF INSTALLATION ON VENTURI METER

as cold-water curves and those for 74 F as hot-water curves.

In Fig. 3, are plotted the curves for 46, 63, and 74 F against throat velocity with the scales exaggerated. It will be noted that all these curves pass through 0.969 at 3 fps. These curves plotted against Reynolds' number are shown in Fig. 4, and do not make a very good showing.

On a number of occasions the author has shown these curves to the Committee and to hydraulic groups of the A.S.C.E., A.S.M.E., and S.P.E.E. The only response ever received has been to question the experimental work rather than to explain the facts. To be in error in the experimental work would mean that the author could not measure 0.15 per ft accurately, which he certainly can.

On a number of occasions, these curves have been placed in the hands of visitors to our laboratory and others who are interested in such matters. Finally a set was given to Dean Goff of this institution, who, after some time, suggested the possibility of heat transfer entering the problem, that is, with cold water being measured and the room temperature at 70 F or thereabouts. The boundary layer at the throat was raised slightly in temperature, thus decreasing the value of μ , decreasing the boundary shear τ_0 , and increasing the discharge for low temperatures.

In order to check this, a wooden box was built around the venturi meter as shown in Fig. 5 and runs were made with a water temperature of 42 F, giving the results shown. The box was drained and the curve immediately rose 0.7 per cent at 3 fps, as indicated. Tests were also conducted with the same arrangement at 53, 56.5, and 61.5 F, as shown in Figs. 6, 7, and 8, respectively.

In Fig. 9, the foregoing tests are plotted against Reynolds' number making quite a creditable showing. The variation of 0.2 per cent must be blamed on the experimenter. To check the effect of installation, tests were made as indicated in Fig. 10. The variations are not extreme.

From the foregoing it would appear that there is a distinct effect of ambient conditions on the coefficient of venturi meters. Unless they are thoroughly lagged during the test and in the permanent installation, they cannot give accurate results under all circumstances.

Discussion

R. W. ANGUS.² Although this paper is brief, it is a distinct contribution to the subject and gives an explanation of what appeared to be an inconsistency in the meter experiments. Evidently the rate of heat transfer through the walls of the meter tube is sufficient to produce erratic coefficients at low throat velocities, while it does not affect those at high throat velocities, presumably because of the time element.

One of the practical applications of the results is that, in the author's meter, the coefficients for different temperatures and for a throat velocity of 10 fps differ by 0.5 per cent. There is about the same difference at 2 fps, in both cases for the meter tested unjacketed; the differences are less than this at intermediate velocities. Inasmuch as it is impossible, in most cases, to jacket meters in service, these errors would seem to be inherent in the use of this instrument but they are likely to be smaller in meters which are larger than the one covered by the author's work. As an effort would be made, wherever possible, to select a meter working with throat velocities corresponding to the flat part of the curve, the error may not be serious in practice, but yet it is there, giving a sense of uncertainty as to the meter indi-

cation. Meters are in common use on waterworks plants where the highest degree of accuracy is usually only necessary when they are being used to measure pump efficiency, in which case the rated pump discharge generally corresponds with the higher throat velocities, the coefficient being constant.

The writer has compared the coefficients in this paper with those given in the Fluid Meters Report.³ The latter shows⁴ that, for the size meter mentioned in the paper, and with water at 68 F, the coefficient has a constant value at over 7 fps throat velocity, while Fig. 2 of the paper shows that, for the meter used with water at 69 F, the coefficient reaches its constant value at about 20 fps. Both of these figures and curves apply to Simplex meters, although one is for an iron tube with bronze throat while the other is for a bronze tube.

The writer has great confidence in the accuracy of the author's work but believes that the high degree of precision he has achieved may cause his results to be misinterpreted. The report³ mentioned bears every evidence of great care in its preparation and must be looked upon as an excellent piece of work. In that report two sets of curves⁵ are given which show many sizes of venturi tubes having a diameter ratio of 0.5. From these curves the coefficients may easily be read to 0.1 per cent. Equation [261]⁶ enables the coefficient to be calculated for any other diameter ratio with the same degree of accuracy; in fact, in the examples given, the coefficients are worked out to 0.01 per cent. The data for these are for the most part based on the experiments of the author. Their accuracy is unquestioned.

The experimental work extended only to tubes for pipes not over 12 in. diam and for a more or less ideal setup of the tubes. However, but slight guidance is given as to what is meant by the proper setup and how it may conveniently be determined whether or not the coefficients will apply to a tube being used in a practical case.

During the summer of 1939, the writer was retained by the City of Toronto, Canada, to conduct tests on eleven new pumping units installed in one of the city pumping stations, and presented the results of these tests in a recent paper.⁷ The guarantees for efficiency were unusually high and the penalties for nonfulfillment of the contract were extremely stringent, particularly on some of the units, so that the writer was obliged to use every possible precaution to secure accuracy.

Discharges were measured with venturi tubes, and the pipes leading the water thereto were as well laid out as possible, the bends being of long radius; in most cases there was a long straight pipe upstream from the tube. In the meter used on one of the pumps, the discharges, computed from the Fluid Meters Report coefficient of 0.9876, were known to be too high. A careful volumetric calibration of the tube was subsequently made, which gave an actual coefficient of 0.9545. There were slight bends upstream from the tube but, ordinarily, these could not have accounted for this great difference. Since the tube had been examined and calipered before erection and again after it had been in use for a time, there appeared to be no other cause for this difference in coefficients than a probable error in the one given by the Fluid Meters Committee, or else a disturbance from the pump.

After discovering this discrepancy, the writer discussed the matter with a number of engineers and found that there were

³ "Theory and Application of Fluid Meters," A.S.M.E. Report of Fluid Meters Committee, Fourth edition, 1937.

⁴ Ibid., Fig. 64, p. 101.

⁵ Ibid., chapter "Discharge Coefficients for Venturi Tubes," Figs. 63 and 64, pp. 100 and 101, respectively.

⁶ Ibid., p. 99.

⁷ "An Improved Technique for Centrifugal-Pump-Efficiency Measurements," by R. W. Angus, Trans. A.S.M.E., vol. 63, January, 1941, pp. 13-19.

² Professor of Mechanical Engineering, University of Toronto, Toronto, Canada. Honorary Member A.S.M.E.

many cases in which the coefficients differed from those recommended by the committee. The writer, therefore, expresses the hope that the Fluid Meters Committee will give this matter further serious consideration, and present to the Society all available evidence in support of its findings, particularly in the case of large meters. Its report should be supplemented by instructions which will assist those using the meters in determining whether the coefficients are likely to apply. While such a device as a straightening vane is usually helpful there is no desire to undergo the expense of installing the device unless it is necessary. Further, the examination of conditions above the meter tube is frequently extremely difficult and will not be undertaken unless there is some definite reason to do so.

In the present state of the art no one knows whether a given setup corresponds to the Fluid Meters Committee's coefficients or not and, further, the report seems to be of theoretical rather than practical value because it does not suggest that actual coefficients are known to deviate from those given, nor does it attempt to quote these coefficients. The tolerance of ± 0.75 per cent⁸ in the coefficient for cast-iron tubes, as given in the report, seems scarcely consistent with the accuracy with which the coefficient is worked out in the examples appearing elsewhere⁹ in the report.

M. M. BORDEN.¹⁰ Before such information can be used for the closer control of venturi coefficients and to limit their tolerances, the matter will require further investigation involving venturi sizes, ratios, effect of roughness, and through a greater range of temperature difference.

The effect of enlarging the area of the throat, because of higher temperatures of a fluid in it, should cause the coefficient to increase. Hence, the full effect of a difference of temperature within and without the tested tubes may be slightly different than indicated.

A plot of the upper and lower coefficients to throat-velocity values for the venturi boxed and unboxed shows a generally narrower zone for the boxed unit. The area, included between such upper and lower limits, is about 10 per cent less for the boxed venturi for regions of 1 to 2 fps and 25 fps throat velocity.

The author's findings suggest the use of completely insulated cold-water venturis by enclosing them in suitable coverings, as is partially done with the covered venturis measuring hot fluids.

E. S. SMITH.¹¹ As one who is now a disinterested observer but formerly an active member of the Fluid Meters Committee, the writer can join a discussion of fluid metering only infrequently and has included in this discussion material which might otherwise be added to the discussion of a current paper¹² on the subject of nozzles. The present discussion gives something of the background of the paper and some material which is only indirectly related to the former bone of contention, i.e., the acceptance of the method of similarity in fluid metering.

Over an extended period, the subcommittee on "similarity," consisted of Messrs. Pardoe, Spitzglass, and the writer, with Messrs. Pigott and Buckingham (deceased) lending moral support on occasion. Messrs. Spitzglass and Pardoe long and spiritedly opposed the acceptance of the Reynolds number as a basis for the correlation of fluid-meter coefficients on the grounds

of unstated limitations of the method of similarity and dimensional homogeneity. However, before his death, Mr. Spitzglass succeeded in making it clear that his acceptance of such correlation required that account be taken of the roughness of the line and/or meter, relative to its diameter, a phenomenon which is sometimes known as the "scale effect." In his present paper the author has likewise succeeded in making it clear that his acceptance awaited only an accounting for an additional factor, i.e., the effect of a different temperature of wall and fluid for water, under the conditions of his tests with water.

Since the acceptance of a useful engineering method is too often delayed by blind advocacy equally with opposition, there is a natural question as to the writer's position. He admittedly made several elementary and simplified statements in an effort to picture clearly the concepts of similarity involved. Some of these statements taken alone might give the impression that the writer was blind to any points such as were stressed by the opponents of the acceptance of this method.

Consequently, it is necessary to point out that a paper,¹³ by the writer in 1923, covered the scale effect¹⁴ in Fig. 2 (which is based on some of the author's values) and elsewhere in the text,¹⁵ while the closure¹⁶ noted that errors result from a difference of the temperature of the fluid from that of the walls of the pipe or meter.

In spite of this broad hint, the author's delay in finding the cause of the errors which his tests included is understandable, since the writer for one did not suspect that this was the cause of inconsistencies in tests which were of the order of errors common in hydraulic testing, possibly including the tests which are noted in the second paragraph of the paper. At one time, the writer checked and found correct everything but the atmosphere in the author's laboratory. He is to be congratulated both for his persistence and the excellence of his tests and to be forgiven for his delay. If he had worked much with fluids other than water, he would certainly have earlier embraced the use of the Reynolds number as the only rational basis for their metering in practice and might have missed finding the cause of the slight error which he now stresses.

His location of this error is a credit to the accuracy of his tests rather than an indication that such error amounts to much. Taking a velocity of 2 fps and the water and air temperatures, respectively, at 50 and 70 F, this error could be caused by a difference of level of only 0.6 ft of water at the stated temperatures in the vertical connecting pipes. Such differences often occur upon a change of rate with usual venturi recording and integrating instruments, with their large wells or bells, which cause a movement of liquid in the pressure pipes which is much larger than the metering head. In controlling flow, transient effects are important and such errors may be of consequence. This less than 1 per cent error would amount to less than 0.002 in. on a chart or integrator in which 0.1 in. corresponds with 1 fps throat velocity, a fact which incidentally shows the importance of precise setting at the lowest operating rate.

If the "box-drained" values of Figs. 5 and 10 of the paper be multiplied by the ratios of water viscosities at the water and air temperatures, the corrected values fall close to the "box-over-flowing" values and constitute a rough check of the Goff theory. Of course at the higher rates, the effect of heat transfer disappears. Possibly a current paper¹⁷ on heat transfer may be useful if the

⁸ Reference (3), p. 128.

⁹ *Ibid.*, p. 104.

¹⁰ Chief Engineer, Simplex Valve & Meter Company, Philadelphia, Pa. Mem. A.S.M.E.

¹¹ Hydraulic Engineer, C. J. Tagliabue Manufacturing Company, Brooklyn, N. Y. Mem. A.S.M.E.

¹² "Discharge Coefficients of Long-Radius Flow Nozzles When Used With Pipe Wall Pressure Taps," by H. S. Bean, S. R. Beitler, and R. E. Sprinkle, Trans. A.S.M.E., vol. 63, 1941, pp. 439-445.

¹³ "The Oil Venturi Meter," by E. S. Smith, Jr., Trans. A.S.M.E., vol. 45, 1923, pp. 67-75.

¹⁴ *Ibid.*, Fig. 2, p. 71.

¹⁵ *Ibid.*, par. 17, p. 71.

¹⁶ *Ibid.*, first par. of closure, p. 75.

¹⁷ "Remarks on the Analogy Between Heat Transfer and Momentum Transfer," by L. M. K. Boelter, R. C. Martinelli, and Finn Jonassen, Trans. A.S.M.E., vol. 63, 1941, pp. 447-455.

generally noneddying character of the flow in the throat be considered. In other words, the stated comparisons in Figs. 5 and 10 indicate that the temperatures of both the water and the wall of the "box-drained" tube are that of the ambient air—an indication that is contrary to the accepted heat-transfer teaching that the wall temperature of a conduit containing running water will approximate that of the water instead of that of the air. An explanation is accordingly requested.

Due to the fact that the heat-transfer coefficients for turbulently flowing water and still air are of different orders, changes of wall temperature from that of the stream are ordinarily insignificant except at low velocities. It should not be difficult for the author to obtain a few temperature and pitot traverses and settle this matter in his closure, instead of relying on speculation which is supported by over-all calibrations only, of which there are many.

Errors are to be expected with uninsulated hot-water meters, supercooled liquid-ammonia meters, and gas meters which are heated to prevent the formation of deposits at the throat. A rise of gas temperature should lead to a negative instead of a positive error, since the viscosity of a gas increases with temperature while that of water then falls.

It cannot be assumed that orifice meters would be free from the Goff error, although such error should be in the opposite direction from that of the venturi. Nozzles contained in the pipe and entirely surrounded by fluid should be practically immune to this error except for large diameter ratios which, like orifices, are sensitive to the upstream velocity distribution. For highly accurate venturi measurements, it is possible to use the heated type of gas venturi with the circulation in the heating chamber provided by the differences of pressure existing in the downstream cone.

No effect is to be expected on the flat portion of the coefficient curve for any differential producer. This general principle was brought out by Swift in his papers,¹⁸ which deal with other factors affecting the correlation of flowmeter coefficients with the Reynolds number.

Since a high value of venturi coefficient exists on the flat portion of the curve and starts to fall off at a relatively high value of the throat velocity at usual temperatures and, hence, of the Reynolds number, such a venturi is relatively sensitive to this error. By roughening the approach curve, as was originally suggested to the writer by Herbert N. Eaton, chief of the National Hydraulic Laboratory, and shortening its radius, the flat portion of the curve can be extended to much lower values of Reynolds' number. Hodgson¹⁹ early reduced the length of the throat cylinder and placed the throat taps at the end of the curved portion of the approach curve to lengthen the flat portion of the coefficient curve. The shortening of the radius of the approach curve was used, e.g., in the German standard nozzles to lengthen the flat part of this curve, although such design also includes a lengthening of the nozzle throat to cause filling of the throat by the re-expansion of the contraction which follows the shorter approach radius.

The writer has tried a number of different combinations of approach-curve radius, length of throat, and roughness, and has obtained a worth-while extension of the flat part of the curve for nozzles. Some of these nozzles were tested by the author and accepted with a 0.25 per cent accuracy guarantee, including their indicating, recording, and integrating instruments on a portion

of the flat part of the coefficient curve. This allowed 0.1 per cent full scale for the author's calibration and installation effects and 0.15 per cent for the instrument from maximum to less than one half of the maximum (which corresponds to within 0.07 per cent of full scale). This accuracy was attained with stock instruments which were special mainly as to the spacing of the dial and chart graduations, working clearances and tolerances, and the setting at the lowest operating rate. The performance of such instruments is fairly comparable with the 0.1 per cent of full scale reading reported with a recently developed, highly special instrument used in steam-turbine traverses. These instruments²⁰ also involved a cam and roller and an integrator wheel-on-a-disk.

Since each diameter-ratio of nozzle has a different "shape" and it seems necessary for the best shapes to be available to all, it is desirable for the A.S.M.E. Fluid Meters Committee to sponsor the necessary tests rather than that the best design become the property of any manufacturer. For nozzles, the design would be essentially the same as for the venturi although the approach may possibly be cut off short of the throat cylinder, as suggested by Witte, so that a slightly conical throat may replace the cylindrical throat of the venturi.

The writer referred his recommendations to the committee, several years before resigning from active work thereon, as a starting point toward such a design and then urged such action. The author's paper revives the importance of a design which is free from even small errors due to the Goff effect. However, a design which is best for cold water under positive pressure may have a much shorter life than the long-radius venturi or nozzle where cavitation exists.

AUTHOR'S CLOSURE

The writer agrees with Professor Angus that the effect of ambient temperature will vary with the size of the venturi meter. It will also vary with the design, paint, roughness, and insulation, if any.

The $8 \times 3\frac{3}{8}$ -in. bronze venturi meter (coefficient 0.99), used in these experiments, although it is of the same proportions, is much smoother than the fifty-seven cast-iron venturi meters used as a basis for the A.S.M.E. curves, from which a value of 0.983 would be obtained. Coefficients of rough venturi meters become flat at much lower throat velocities. Coefficients of venturi meters should not contain more than three significant figures, although Prof. Angus uses four.

Some idea of the "proper setup" may be obtained by reference to a previous paper²¹ by the author.

The venturi meter at Toronto, which gave a coefficient of 0.9545 instead of 0.9876 or 3.3 per cent low, was one of eleven meters. All the others gave satisfactory results, using the A.S.M.E. coefficient values (in which the author is not disinterested). Prof. Angus examined and calipered the meter and found the pressure taps in good condition. Under these circumstances, the only thing which could lower the coefficient 3.3 per cent would be a vortex started in the centrifugal pump. That this could produce the result is evident from Fig. 44²² of the author's previous paper. The author has found it advantageous in all cases where a vortex may form to use cross straightening vanes 4 diam in length ahead of the venturi meter. Until this is done or an exploration made ahead of the Toronto meter, this matter cannot be considered settled. The author cannot believe a 42

¹⁸ "Orifice Flow as Affected by Viscosity and Capillary," by H. W. Swift, *Philosophical Magazine*, series 7, vol. 2, 1926, pp. 852-875; "Operational Factors in Orifice Flow," vol. 5, 1928, pp. 1-17; "The Calibration of an Orifice," vol. 8, 1929, pp. 409-435.

¹⁹ "The Measurement of the Flow of Gases and Liquids by Means of Orifices, Nozzles, and Venturi Tubes," by J. L. Hodgson, *World Engineering Congress*, vol. 4, part 2, Tokio, 1929, p. 113, Fig. 10.

²⁰ "Automatic Integrating Pressure-Traversal Recorder for Study of Flow Phenomena in Steam-Turbine Nozzles and Buckets," by H. Kraft and T. M. Berry, *Trans. A.S.M.E.*, vol. 62, Aug., 1940, pp. 479-488.

²¹ "The Effect of Installation on the Coefficients of Venturi Meters," by W. S. Pardoe, *Trans. A.S.M.E.*, vol. 58, 1936, pp. 677-684.

²² Reference (21), p. 683.

× 27-in. venturi meter, correctly constructed and without vortex flow, could possibly give such a low coefficient.

Mr. Borden suggests a great deal of additional experimental work. The author does not propose to do any beyond establishing the fact of an effect of ambient on the coefficients of flow meters generally.

Mr. Smith has quite misinterpreted the paper and has forgotten our "bone of contention," i.e., that Reynolds' number is



FIG. 11 EFFECT OF AMBIENT TEMPERATURE ON COEFFICIENTS FOR FLOW NOZZLE

(A.S.M.E. flow nozzle 8:300:1; 8.071 × 3.0012 in.; temperature of water 41 F; temperature of air 67.5 F.)

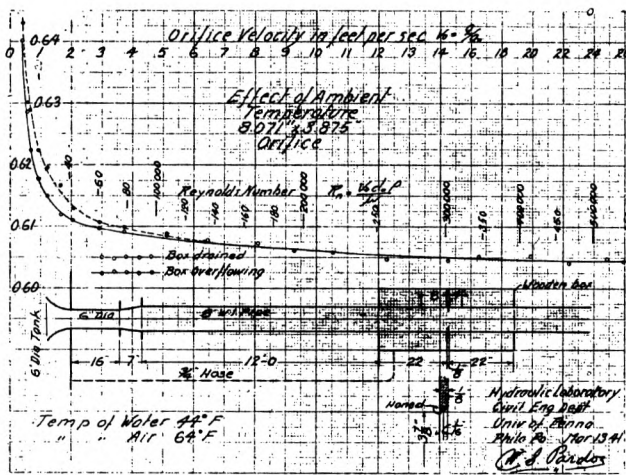


FIG. 12 EFFECT OF AMBIENT TEMPERATURE ON COEFFICIENTS FOR PIPE ORIFICE

(Size 8.071 × 3.875 in.; temperature of water 44 F; temperature of air 64 F.)

not a satisfactory criterion for plotting the coefficients of venturi meters as usually constructed and installed, it cannot be used on the curved part of the coefficient curve, and has no meaning on the flat part. The errors inside the metering range of manufacturers' meters have already been mentioned and are considerable. The author now includes flow nozzles and pipe orifices as being similarly affected, as indicated by the curves in Figs. 11 and 12 of this closure. Mr. Smith will have to revise his ideas with respect to the orifice and also his very "rough" check of the Goff theory. He suggests that "it should not be difficult to get a few temperature and pitot-tube traverses." The author must disagree and believes this almost impossible as we are dealing not with the laminar boundary layer but with the sublamina boundary layer, at very low velocities. If Mr. Smith has in mind a pitot tube to measure the wall velocity (if any), there are many individuals who will be very much interested in learning of it.

For a number of years, the author kept up a correspondence

with Dr. Edgar Buckingham, sending him the original curves as obtained. He expressed the belief that "we were leaving out some factor," but at no time suggested heat transfer, nor did any other member of the Committee, although all of them were kept continually aware of the unsatisfactory results obtained with water. The effect is too complicated to evaluate from any reasonable number of experiments depending upon, as it does, the specific heats of the fluids, the design, the roughness, the materials of construction, the paint, the insulation, and probably other things. In other words, life is too short for this busy "hexagenarian" even to start.

During the discussion of the paper, Messrs. Sprengle, Beitler, and Bean suggested that the flow nozzle was not affected by ambient temperature. Herewith is shown test results of an A.S.M.E. flow nozzle 8:300:1, Fig. 11, indicating that they are not immune. Fig. 12, for an 8.071 × 3.875-in. orifice also shows that it is affected in quite a similar manner, and not the reverse as suggested.

The coefficient of a flowmeter may be expressed

$$C = \sqrt{\frac{1 - \beta^4}{\alpha_2 - \alpha_1 \beta^4 + k}}$$

in which

$$\beta = d_2/d_1$$

$$k = \text{coefficient of loss in } hf = k \frac{\bar{V}_2^2}{2g}$$

$$\alpha = \text{ratio between actual kinetic energy per pound and kinetic energy per pound or } \bar{V}^2/2g$$

$$\alpha_2 = 1.00$$

If the main velocity traverse given by the seventh-root law for smooth pipe

$$V = V_{\max} \left(\frac{y}{r_0} \right)^{1/7}$$

$$\text{Pipe factor} = 0.817$$

$$\alpha_1 = 1.056$$

$$\therefore k = \frac{1 - \beta^4}{C^2} - (1 - 1.056 \beta^4)$$

Take an 8.071 × 3.0012-in. flow nozzle at 2 fps throat velocity

$$C_d = 0.9662 \text{ box drained}$$

$$C_o = 0.9606 \text{ box overflowing}$$

$$\beta = 0.37184 \quad \beta^4 = 0.019118$$

$$k_d = \frac{1 - 0.019118}{0.9662^2} - (1 - 1.056 \times 0.019118) = 0.0715$$

$$k_o = \frac{1 - 0.019118}{0.9606^2} - (1 - 1.056 \times 0.019118) = 0.0840$$

As

$$\tau = \mu \frac{dv}{dy} = f \frac{\rho \bar{V}^2}{8} = w \frac{f \bar{V}^2}{4 \cdot 2g} = wk \frac{\bar{V}^2}{2g}$$

therefore

$$\frac{k_d}{k_o} = \frac{\tau_d}{\tau_o} = \frac{\mu_d}{\mu_o} \quad \left\{ \begin{array}{l} \text{is assumed constant as} \\ \text{velocity is constant, and} \\ \text{pipe factor} = 1 \text{ approxi-} \\ \text{mately} \end{array} \right.$$

and

$$\mu_d = \mu_0 \frac{k_d}{k_0} = 0.0000316 \frac{0.0715}{0.0840} = 0.0000269$$

The corresponding temperature is 51 F, i.e., the inside wall of the flow nozzle would have to be 10 F above the water temperature. As the velocity at the wall is zero, this must be the water temperature at the wall. It does not seem that this is impossible with a temperature gradient of 67.5 — 41 or 26.5 F.

Similar computations for other velocities are given in Table 1.

That is, the difference in temperature between the water and the inside wall is inversely as the velocity, which seems quite reasonable.

The author desires to thank those who participated in either the written or oral discussion. Many valuable suggestions were made

TABLE 1

Throat velocity, fps	Wall temperature, F	Wall temperature —41 F
1	52	11
2	51	10
3	50	9
4	49.1	8.1
5	48.1	7.1
6	47	6
8	44.8	3.8
10	42.7	1.7

as to how to proceed, and a tremendous amount of additional experimental work was outlined for the author, which he regrets he will be unable to do. He is not particularly concerned with the physicist's explanation of the fact, but is tremendously concerned with the fact and how to get around it.